Der beiwert n im eisenbetonbau, (The "n" Value in Reinforced Concrete Design), Beton and Eisen (Berlin), April 7, 1937

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The invitation from Dr. Fritz Emperger to discuss the problem of the "n" value in reinforced concrete design is a most welcome opportunity for emphasizing the necessity for more experimental research work in this particular field. Lately European technical publications and in particular "Beton und Eisen" have presented numerous viewpoints regarding the proper "n" value to use when designing reinforced concrete members for given factors of safety. Whatever experimental evidences are available indicate that the "n" values which have been used in the past when designing for given permissible stresses may be of little value when designing on the basis of a given factor of safety; and since the factor of safety is more and more being recognized as the proper design criterion, it is high time that some serious considerations be given to the method of calculating accurately the ultimate strength of reinforced concrete members.

For reinforced concrete columns experimental results have shown that the summation method of computing the strength is very satisfactory. That means that the "n" value has no significance in reinforced concrete column design.

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The strength of a reinforced concrete column with no spiral reinforcement is given by:

\[ F = 0.85 f'_c A_c + f_{sy} A_s \]

and with spiral reinforcement:

\[ F = 0.85 f'_c A_c + f_{sy} A_s + k f'_{sy} A'_s \]

where

- \( F \) = strength of column
- \( f'_c \) = strength of 6"x12" control cylinder
- \( A_c \) = area of concrete in column
- \( f_{sy} \) = yield point stress of longitudinal reinforcement
- \( A_s \) = area of longitudinal reinforcement
- \( k \) = effectiveness factor for spiral reinforcement
  (may be assumed approximately 2.0)
- \( f'_{sy} \) = yield-point stress of spiral reinforcement
- \( A'_s \) = area of spiral reinforcement

Applying proper factors of safety to these strength values will give the most logical basis for rational design of reinforced concrete columns.

However, for members subjected to flexural conditions such as beams and slabs, the "n" value has until recently been considered a very necessary element in the design. The location of the neutral axis has generally been determined by assuming straight line stress distribution in the compressive portion of the concrete. While the straight line distribution has been the prevailing practice, Professor A. N. Talbot already in 1905 pointed out that at high stresses the
stress distribution in the concrete became essentially parabolic (see University of Illinois Bulletin No. 4). Dr. Fritz Emperger in his excellent paper DER BEIWERT "n" published in the October 5, 1936 issue of BETON UND EISEN recommends a trapezoidal stress distribution in the concrete at failure. In the United States the parabolic distribution has been considered by several investigations since Talbott's first proposal, but little progress has been made. Various values of "n" has been given by different building codes and the most recent recommendation is found in the empirical formula:

\[ n = \frac{30,000}{f'c} \]

However, experimental results have indicated that these "n" values are not well founded. In order to obtain information on the actual strength of reinforced concrete beams when failure occurred in the concrete an extensive investigation was carried out at Lehigh University a few years ago. The results of this investigation was published in a paper entitled COMPRESSIVE STRENGTH OF CONCRETE IN FLEXURE AS DETERMINED FROM TESTS OF REINFORCED BEAMS by W. A. Slater and Inge Lyse (Proceedings of the American Concrete Institute, Vol.26, 1930, p. 631). Five groups of beams in this investigation had strengths of concrete varying from about 1400 lb per sq. inch to 5800 lb per sq. inch. The effect of the strength of the concrete on the obtained maximum resisting moment of the beams is shown in Fig. 1. It is noted that except for the very lean concrete (1400 lb per sq. in.) the maximum moment at failure increased very uniformly with the increase in the strength of the concrete, indicating that the position of the neutral axis remained stationary. Parabolic stress distribution was considered in the study of these results and the formulas for the location of the neutral axis and the distance between the compressive
and tensile centers are given in the paper. The ratio of the distance between the centers of pressure to the effective depth of the beam is given by:

\[ j = 1 - \frac{r+1}{2(r+2)} \cdot k \]

where \( r \) is the power of the parabola and \( k \) is the ratio of the distance from the compressive surface of the concrete to the neutral axis to the effective depth of the beam. If the neutral axis is assumed at 1/2 of the effective depth, that is \( k = 1/2 \), the degree of the parabola which will give computed values in harmony with the experimental results may readily be computed. In Fig. 1 the dotted straight line gives the computed moments at failure when a fifth degree parabola is used for the stress distribution in the concrete. The design formulas for this condition become:

\[ j = 1 - \frac{5+1}{2(5+2)} \cdot \frac{1}{2} = 1 - \frac{3}{14} = \frac{11}{14} \]

\[ M' = \frac{5/6 \cdot 11/14 \cdot b \cdot d^2 \cdot f'_c}{b \cdot d^2 \cdot f'_c} = 55/168 \cdot b \cdot d^2 \cdot f'_c \text{ or about } \frac{1}{3} b \cdot d^2 \cdot f'_c \]  \( \text{(1)} \)

and \( f_s = \frac{M'}{11/14 \cdot d \cdot A_s} \)  \( \text{(2)} \)

The maximum moments thus computed agree remarkably well with the moments obtained from the test specimens. In Table 8 of the paper referred to the observed strains in the reinforcement are given for loads very close to the maximum. In the following tabulation the stresses in the steel computed by the use of equation (2) are given together with the stresses obtained when the observed strains are multiplied by a modulus of elasticity of 29,000,000 lb per sq. inch, which is generally used for steel.
Summarizing the work in the United States it must be admitted that very little progress has been made and little data are available for the development of a more scientific method of computing the strength of flexural members. Extensive research investigations in this field must therefore be undertaken if we are to achieve rational and economic design of reinforced-concrete structures. Since the problem is of fundamental importance to the whole concrete industry, international cooperation in its solution is very much needed. It is therefore hoped that international organizations interested in this field will arrange for the necessary meeting for a thorough discussion of the question of the "n" value in reinforced concrete design.