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Welded Continuous Frames and Their Components

ON THE APPLICATION OF PLASTIC DESIGN

by

Lynn S. Beedle

Fritz Engineering Laboratory Report No. 205.70
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ENGINEERS and research workers have been stimulated to study the plastic strength of steel structures and its application to design for three principal reasons: (a) it has a more logical design basis, (b) it is more economical in the use of steel, and (c) it represents a substantial savings of time in the design office.

The purpose of these remarks is to show how plastic design has been applied in the United States and to indicate the extent to which the advantages claimed for it have, in fact, influenced its use. A further purpose is to indicate possible future applications of plastic design. Some of the important research problems engendered will be given and, for some of these, the current theoretical and experimental approaches will be discussed.

The methods of designing nearly all steel structures in the past were based upon an allowable stress which incorporated a factor of safety against the elastic limit.
Plastic design, on the other hand, represents the utilization of the reserve strength manifested during deformation beyond the elastic limit. Figure 1 illustrates the difference in the two approaches. The ultimate load-carrying capacity of the simply supported beam is but little above the yield load \( P_y \). Therefore, the working load \( P_w \) is a suitable design basis. On the other hand, the load-deflection curve of the fixed-ended beam confirms that failure does not correspond to attainment of the elastic limit at the ends. There is a considerable reserve of load-carrying capacity beyond the yield load \( P_y \). The ultimate load \( P_u \) is not reached until yield zones (plastic hinges) have developed at the ends and at the center. It is through plastic design that this reserve of strength beyond the elastic limit is utilized. The design basis becomes the ultimate load \( P_u \).

The first application of plastic design could almost be called "unconscious". There are at least a dozen ways in which ductility of steel has been counted upon in elastic design - knowingly or not. (1) In the
first place, certain factors are neglected because of the compensating effect of ductility; in the second place, working stresses have frequently been revised because the "normal" value was too conservative.

Figure 2 illustrates one such example from present design practice and is concerned with the permissible stresses in a round pin. The curve shows on a non-dimensional basis the moment-rotation characteristics of a round pin and a wide-flange beam. (Sketches at $\phi/\phi_y = 2$ show the yielded portions of the two cross sections as shaded.) The maximum bending strength of the wide-flange beam is $1.14 \, M_y$, whereas that of the pin is $1.70 \, M_y$. According to the AISC, the permissible design stresses are 20 ksi for the WF beam and 30 ksi for the pin. Although the stresses are different, the tabulations on the figure show that the true load factor of safety for each case is identical. While somewhat of a coincidence, this exact agreement is indicative of the influence of long years of experience which has resulted in different permissible working stresses for various geometrical conditions.
As shown in Fig. 3, the first "conscious" application of plastic design was in Hungary in 1914. A period of research followed. Then in 1939 plastic design saw considerable practical application in the design of shelters for protecting families against bomb blast. One million, two-hundred thousand were built and documented evidence shows that they performed as predicted, deforming plastically to absorb impact but remaining intact to afford the necessary protection.\(^{(2)}\)

British Standard Specification 449, as issued in 1948, contained a clause permitting the use of plastic design. Four years later (1952) the first building in England was erected according to the plastic method. From that time on, the progress was very rapid and by 1958 more than 600 structures had been designed by the plastic method. Although they were mostly industrial buildings, they also included several four-and five-story structures.\(^{(3)}\)

In this country, the introduction of a 20% increase in allowable stress at points of interior support was a partial recognition of reserve plastic strength. This
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was adopted by the AISC in 1946.

The first building to be designed by the plastic method on this continent was in Canada about 10 years later.\(^{(4)}\) It was a two-story frame with beams continuous over six spans. A few months later in 1957 a warehouse was erected in Sioux Falls, South Dakota.\(^{(5)}\)

The AISC Specification for plastic design was adopted in December 1958 but by this time at least a score of plastically-designed structures had been built in spite of the out-dated codes. The number has now risen to about 175.

The rapid acceptance in this country of a design concept introduced on the continent of Europe almost 40 years ago\(^{(6)}\) is due to a number of factors. One is certainly the competitive nature of the construction industry. Not the least role, however, has been that of educational programs designed to inform educators and engineers alike of the results of structural research. In Fig. 4 the program of education and research is shown on a time-scale base.
Research on this subject commenced at Lehigh University in 1946, following a suggestion made in 1945 by the Welding Research Council concerning the nature of the research which it wished to resume. In addition to WRC, the program was supported by the Navy Department, the American Iron and Steel Institute, and the American Institute of Steel Construction. At about the same time, work was starting at Brown University under sponsorship of the Navy Department on a critical survey of the mathematical theory of plasticity. This work, in part, was an outgrowth of problems faced by the Bureau of Ships in designing the underwater protection systems in Naval vessels to absorb underwater explosions through plastic deformation. Both of these investigations, of course, took cognizance of the valuable work of Prof. J. F. Baker and his staff at Cambridge University.

In 1940 Prof. Vandenberg presented his paper on limit design to the ASCE (7) and since that time both that society and the American Welding Society, the SESA, the AISC, AISI, and ASEE as well as others have scheduled technical sessions where research results and evaluations of plastic design have been presented.
Symposia at Brown University, with ONR support, have helped a great deal in bringing together research workers. The first of these was in 1948. In 1955 a "summer course" at Lehigh University presented plastic design in a form suitable for engineering educators. The AISC National Engineering conference at Lehigh University in 1956 was devoted entirely to plastic design, the material being presented primarily for the engineer. (8) AISC - sponsored "Regional Conferences" followed in the years 1956-1958. The current AISC lecture series, designed for practicing engineers, commenced last year. It has been presented in 37 major cities throughout the country with a total attendance of about 7000 engineers.
CODES AND SPECIFICATIONS

As shown in Fig. 4, the first code or specification in North America was that which was approved by the Canadian Standards Association.

In 1956 the ASCE Committee on Plasticity Related to Design took action to join with an existing Welding Research Council Committee for the purpose of preparing a "Commentary" on Plastic Design. This Commentary, now published, (9) demonstrates the applicability of plastic analysis to design of structural steel beams and frames. Theoretical considerations involved in the plastic theory and in certain secondary design problems are given. Experimental verification is provided. Approximations in the form of "design guides" are suggested.

The complete table of contents of the Commentary is shown in Fig. 5. The first of the series of seven progress reports was published in July, 1959, the final two installments appearing in the April, 1960, Journal of the ASCE Engineering Mechanics Division.
On December 4, 1958, the first specification in the United States for plastic design was issued by the AISC.\(^{(10)}\) Actually the use of the concept had been approved two years earlier based on the 1956 AISC National Engineering Conference Proceedings.\(^{(8)}\) This was followed in January 1959 with the issuing of an AISC manual prepared by T. R. Higgins and E. R. Estes to assist the practicing engineer in the use of plastic design for the solution of practical structural engineering problems.\(^{(11)}\)

In addition to the AISC Specification (and based upon it), the four major national building codes of the United States have now adopted plastic design. These are the Building Officials Conference of America, the International Building Officials Conference, the Southern Building Congress, and the National Board of Fire Underwriters. Most local codes have adopted plastic design, particularly in the southeast where its application is commonplace.

With regard to federal agencies, their procedures permit the use of plastic analysis for protective
construction. As yet, most of their specifications do not explicitly provide for its use in ordinary construction, but it is anticipated that revisions currently under way will permit further use of plastic design.

In addition to progress in England, (12) Fig. 6 shows that codes and specifications throughout the world are gradually being revised to incorporate this method. As indicated, there are at least nine countries in which plastic design is either used, permitted, or about to be authorized.
CURRENT PRACTICE

BUILDINGS

As described in a separate paper \(^{(11)}\), plastic design has seen extensive use in England. It is reported that nearly half of all new designs of single-span gabled frames are according to the plastic method and that some companies do all of their single-story portal frames by plastic design.

It is clear from Fig. 6 that the number of plastically-designed structures in the United States (about 175) doesn't begin to approach those in the United Kingdom. However, according to Fig. 3, the rate of progress during the first few years seems to compare favorably.

Figure 7 gives an indication of the types of building frames that have been built according to plastic design. It covers continuous beams on column supports, rectangular rigid frames, gabled frames (both single-span and multi-span), two-story structures, multi-story braced frames, and others with somewhat unusual geometry.
The predominant use of plastic design in the United States has been for continuous beams and rectangular and gabled single-span frames.

There follows a brief description of some of the most interesting of the designs that have been completed up to the present time. It supplements that which was given in Ref. 13.

The Horace Mann School gymnasium shown under construction in Fig. 8 is typical of numerous plastically-designed structures. It was designed by the Leo A. Daly Company. The span is 97 feet with flat roof. Figure 9 shows the effective support given to the knee and to the compression flange in the zone that would become plastic as ultimate load was approached.

In the West-Side High School in Omaha one of the reasons given for the use of plastic design was that it eliminated the need for a haunch that would have been required by elastic design. The parallel-flange rolled members (36WF170) permitted a better architectural treatment. The engineer could proceed with the confidence
that his design not only would satisfy the architect, but would also be economical of material.

Located on President's Island in Memphis, Tenn., the 92-foot rigid frame shown in Fig. 10 serves as a warehouse. 27WF76 and 24WF94 shapes were used, the structure being designed for future possible additions. The plastic design also considered the use of a crane.

Figure 11 shows an eight-story apartment building in Canada the beams of which were plastically designed. The span between columns (which were elastically-designed) was 39 feet, with cantilever extensions. X-type bracing was used to prevent sway. The cost savings was reported as about $10,000 when compared with a reinforced concrete design.

The tire plant of the Gates Rubber Company in Nashville, Tenn., constitutes one of the large plastically designed structures. Fig. 12. It was designed by the Rust Engineering Company and is a production and warehouse building. The six gabled frames are continuous with spans of 60-ft. each. The column heights are 31 feet and the frames are spaced on 25-foot centers. The
360' x 975' building has a floor area of about 350,000 sq. ft. The beams are 21WF62 shapes. 8WF31 shapes were used for the interior columns. The purlins were also designed plastically, being continuous over two spans.

Three plastically-designed buildings were recently completed for the Safeway Distribution Center in Omaha. The largest building, 473 feet by 450 feet, utilized continuous beams on struts. A simple-beam design would have required 982 tons of steel. The plastic design was completed requiring 841 tons, representing a 14% weight saving. The saving in cost was $24,500.

A considerable number of the Dixisteel buildings that make use of plastic design have been erected. Figure 13 shows one of these, the Coca Cola plant in Biloxi, Mississippi. Many of these frames make use of tapered sections and employ both longitudinal and vertical stiffeners to overcome web depth-to-thickness limitations. The main advantage of plastic analysis has been the ease of design and optimum usage of the tapered section.

The Church of Christ the King in Sioux Falls was designed with a frame of unusual shape - an inverted
keystone 54 feet in height with a span of 50 feet at the base. An artist's rendering is shown in Fig. 14.

OTHER STRUCTURES

The application of plastic design to structures other than buildings has been more extensive than might otherwise be realized. Unfortunately, it is not possible to give as complete a documentation because many of the applications have involved protective construction.

The importance of the ductility of steel in the design of ships has long been recognized. The design of bulkheads to withstand explosions is one such illustration. Further applications in the analysis of ship structures have been described in Refs. 17 and 18.

Plastic analysis is being used by various agencies in the Department of Defense. In some instances, it has been used for the design of protective construction where plastic deformations must be accepted. It has been indicated that the forthcoming design manual of the Bureau of Yards and Docks will contain information
on this method. The Defense Atomic Support Agency has applied it for the design of test structures. Other agencies make considerable use of plastic analysis for estimating the strength of existing structures.
SUMMARY OF APPLICATIONS

During the period in which researchers investigated the application of the plastic method for design, and during the time that the method was explained and described to the profession, the following advantages of plastic design and elastic design were stressed:

1) Saving of material
2) Saving of design time
3) Uniform load factor (rational design basis)
4) Ease of investigating a number of design possibilities in a short time

It is interesting to see how these claims were realized by designers. Figure 15 outlines the factors that were decisive for the choice of plastic design over other methods. These reasons are:

1) Weight saving has been a prime consideration in some instances, the possible variation ranging from nothing to 35%. A figure of 15% is common.

2) A saving in overall cost has also prompted its use. As determined on the basis of actual bids
for different design and construction methods, plastic design for one building saved $24,500.

3) Through plastic design, the elimination of haunches in a school in Omaha not only simplified the fabrication but also permitted better architectural treatment.

4) In another instance, a completed elastic design was found to be inadequate. Reinforcement plates were specified for the haunches, permitting the structure then to meet the necessary conditions for plastic design.

5) Another elastic design was adequate to meet the originally specified loading conditions; but plastic analysis showed that the same members were adequate for a more severe loading condition that was later specified.

6) During the 1959 steel strike, work could proceed on a structure when plastic design was authorized, permitting the use of a shape that otherwise would have meant a deficient elastic design.
7) The first plastically designed structure in North America proceeded because a larger shape required by elastic design could not be procured within the necessary time limit.

8) The factor influencing the first major application was energy absorption. A world-war II shelter for night-time occupancy was designed to absorb blast impact through plastic deformation. In more recent years, numerous designs to withstand atomic blast have been based on the same concept. The military forces are also able to make use of plastic analysis to evaluate explosive energy required to demolish test structures.

9) It is difficult to show that it was used solely because design time was saved, but strong indirect evidence is available. It is quite difficult to collect much information on the comparative weights of plastic designs and indeterminate elastic designs. Often the answer is, "we haven't taken time to make the
elastic analysis". Actually what is probably happening is that engineers are designing continuous structures plastically in about the same time that they were doing "simple beam" designs.

10) As engineers come to realize the potentialities of this new method they will make more and more use of it because it is a method of design, not analysis. This has been effectively pointed out in Ref. 19 from which the following is quoted:

"...the plastic method enables the designer to dictate to the structure the precise manner in which he wants it to resist the external forces, to tell the structure how much of the total static moment he wants to be resisted at one section, and how much at another. It is this which makes the plastic method primarily one of design, while the elastic method with its limited freedom of action is primarily one of analysis.

It is likely that this new freedom of shaping structures optimally, in regard to economy in some cases, in regard to aesthetics in others, will be appreciated only gradually as designers, through use, become aware of the new potentialities of this approach."
Thus the presumed advantages of plastic design have in fact influenced its use in design. At the same time, numerous other factors have stimulated the progress of plastic design, not the least of which has been necessity.

One interesting feature has been the function for which plastically-designed structures have been erected. The first predictions were that warehouses and military structures would be appropriate applications of this technique. Quite the reverse trend has been observed, as is shown in Fig. 16. The greatest single percentage is schools and gymnasiums (30%). Military structures, warehouses, and industrial buildings constitute only 30% of the total. The majority (70%) are for personnel.

In November of 1959 about 75 plastically-designed structures had been erected in 13 states. The present (April, 1960) estimate of 175 includes structures in 27 different states as seen in Fig. 17.
In addition to the application of plastic design for one- and two-story building frames, some of the additional opportunities that exist are in structures such as multi-story frames, trusses (Vierendeel and ordinary trusses), bridges, arches and rings, and ships.

For each of these structures there are some unsolved problems of importance to American designers. Of course, each design problem engenders one or more research problem, and these are outlined in Fig. 18. In some cases, the solution is complete or nearly in hand. In other cases, research has not yet started. Mention will now be made of current work on some of the research problems where an attempt is being made to answer specific design questions. Some of the preliminary results will be mentioned. Most of these problems are concerned with the design of multi-story frames.

MULTI-STORY FRAMES

Plastic design already has had a limited start in the case of a modest number of stories, and such structures
offer challenging opportunities. A complete solution to the problem of the plastic design of multi-story braced frames requires a study of the proper distribution of moments to the columns and the loading conditions to cause the most severe loading case on the columns. Currently at Lehigh University suggested design procedures are being developed on the basis that the beams will be designed plastically and that the columns will be designed either elastically or by the provisions of the AISC Plastic Design Specification. Bracing against sidesway would be designed by conventional methods.

The next step would be to cover the design of columns in braced frames using the results of the restrained column study (see below). This would be followed by a consideration of practical design methods for multi-story frames in which cross-bracing against sidesway is omitted.

**USE OF HIGH-STRENGTH BOLTS**

More and more high-strength bolts are replacing rivets, particularly in field connections. However, up
to the present time they have been installed in accordance with a specification which requires that as many bolts be used as otherwise would be specified for rivets. It is known, of course, that A325 bolts are much stronger than rivets, and for some time research has been continuing at Lehigh University in an attempt to make more effective use of the new fastener. (20) Quite recently the Research Council on Riveted and Bolted Structural Joints adopted a new specification that is based, in part, upon this research. One interesting feature of these studies is that the bolt, in addition to its greater strength, also can exhibit a deformation capacity that is comparable to that of the rivet.

The AISC is sponsoring research at Cornell University into the use of high-strength bolts in moment connections. One of the specific problems will be to develop methods of design to assure that such connections can transmit the plastic moment of the beams joined.

DEVELOPMENT OF COMPOSITE ACTION

Previous research on the development of composite action between a concrete slab and a steel beam has been
directed primarily towards bridge construction. It appears, however, that the resulting requirements for shear connectors are too conservative when considered for statically loaded building floors. The present effort is aimed at developing the full plastic moment of the total cross section and indicating how the shear connectors must be distributed and spaced. Because of the lighter beams to be used in building floors than in bridge construction, it is desirable to find out how much interaction is created by natural bond and friction alone in the absence of any shear devices. Also of interest are the strength and deformation characteristics of shear devices and the influence of slip on the load-deflection curve of a composite beam. Current research at Lehigh is also exploring the likelihood of increasing the permissible loads on composite beams on the basis of their demonstrated plastic action.

Figure 19 shows in the lower curve the moment-deformation characteristics of a WF shape. The upper curve is for a composite beam with its greater margin of reserve strength beyond the elastic limit (about 50%).
Thus there is a potential increase in working load capacity of 1/3, even for a simple beam. A WF beam working at 0.61 times the yield value reflects a load factor of 1.85. If this same load factor is used for a composite beam, the allowable working moment would be increased to 0.81 $M_y$.

Figure 20 shows the results of tests with connectors as required by current bridge specifications (Test B1-T1), as required by a prediction based on ultimate strength (B3-T1), and no connectors (B2-T1). Quite evidently the number of connectors now specified is uneconomical; fewer connectors are necessary to develop the full plastic moment. (21) In fact, the results so far indicate that the number of shear devices in composite beams for buildings may be reduced to as few as one-half of the number which would be required if present recommendations for shear devices in composite beams for bridges were followed.

FRAME STABILITY

If side-sway is not prevented, a rigid frame may buckle as a whole before the plastic mechanism is formed.
Figure 21 shows the three modes of failure that may occur, depending on the loading. If all of the load acts symmetrically on the columns there is no lateral deflection until bifurcation occurs; if symmetrical loads are also applied to the beam, then small lateral deflections also develop prior to bifurcation; and if lateral load is applied from the beginning, the point of frame instability is the maximum load on the curve.

A considerable amount of research work has been done to obtain elastic solutions, but little information is available for determining the stability of partially yielded frames. The problem is now being studied at Lehigh University and elsewhere.\(^9\)

In the meanwhile, an interim design guide for single-story frames was obtained by considering the behavior of an analogous frame (Fig. 22). The plastic hinge which would be unloading at failure is assumed to behave elastically. A real hinge is substituted at the other column. As a result of the analysis, the dashed line in Fig. 22 was selected as the limit below which frame instability would not be a problem. There is experimental
evidence to support this conclusion.

In addition to this, it has been shown that cladding and partial column base fixity offer substantial resistance to frame instability\(^{(22)}\)

Currently, theoretical and experimental work is under way at Lehigh for loading conditions not included within the present design guide. An attempt is being made to predict the inelastic buckling load. Figure 23 shows the results of experiments on small frames (50-in. span) using built-up members. It shows the agreement with the elastic solution (dashed) and the reduction in the inelastic region. At \(P/P_y = 0.4\) the test points above the theoretical elastic and plastic lines show the remarkable increase in buckling strength due to the partial base fixity supplied by a minimum size column base plate on a foundation resting on "minimum" resistance soil.

The trend is encouraging. After completing these studies new recommendations would be expected to show how the proportion frames in which instability would
not be a problem, or in which the column size could be adjusted to assure attainment of the desired ultimate load.

**COLUMNS**

It is significant that the very first work done on the research project at Lehigh had to do with columns in rigid frames. Many problems still remain in spite of the intensive research effort at Fritz Laboratory and elsewhere. However substantial progress has been made; the time is approaching when column design procedures will be available that take into account not only inelastic behavior of isolated members (23), but also elastic and inelastic restraint afforded by adjoining beams (24), and the influence of lateral-torsional buckling (25).

A remarkable recent development has to do with rotation capacity -- the ability of a column to support the modified hinge moment through sufficient hinge rotation so that other necessary hinges will form. Figure 24 shows an experimental M-Θ curve at \( P/P_y = 0.33 \), a value higher than that commonly encountered in one-
and two-story structures. The dashed line shows the predicted curve \((P/P_y = 0.3)\) obtained as an outgrowth of the current study of restrained columns. The theory has also been confirmed for other slenderness and axial load ratios, and this part of the research problem can be considered as solved.

**BRIDGES**

One is accustomed to think of bridge behavior in terms of repeated loading. Fatigue is frequently a factor in certain connections for some bridge members. However, there are undoubtedly conditions under which static load-carrying capacity is the controlling design criterion. In these instances the possibility of plastic design should be examined. Indeed, one such application already exists in Canada.\(^{(26)}\)

In order to define the conditions for which the loading may be considered as static, more studies are needed of the behavior of structures under variable repeated loading. Some of the necessary studies are underway at the University of California (Berkeley). Computer solutions are being developed which make
possible examination of "incremental collapse" loads. It is planned to study many practical types of structures and also those of unusual geometry making use of a special repeated load machine to accommodate large reversals of strain.

For plate girders, web buckling is often thought to be a limitation upon useful capacity. Current research at Fritz Laboratory shows that such is not the case.\(^{(27)}\) The application of plastic design to plate girders depends rather on the behavior of the compression flange.

Figure 25 shows the behavior (on a load-deflection basis) of a number of plate girders tested in the program. The behavior of girders G3 and G5 (slenderness ratios 185 and 388, respectively) is especially interesting because it shows significant deformation capacity, an item whose importance is generally overlooked because it does not affect conventional designs. If a section of a statically indeterminate plate girder were able to sustain a curvature well beyond that which produces first yielding, it is possible that redistribution of moments would take place. The behavior of girders like
SHIP STRUCTURES

A large naval vessel is one of the most complicated of all steel structures. It is natural, therefore, to explore the applicability of plastic design for ship structures.

Once the loading is known, the design of the overall ship girder is reduced to that of a statically determinate beam. Plastic design would not have a significant advantage in such a case, although some time would be saved in computing the strength of the cross section and there would be a modest advantage through the utilization of the shape factor.

On the other hand, just as in the case of buildings, the real advantage for plastic design of ships lies in designing the indeterminate parts of the frame. Two examples are:

a) The transverse frames of a ship are rigid indeterminate frames which are amenable to
plastic analysis.

b) Grillage supports for the load-carrying decks are highly redundant and can be analyzed by the plastic method. In fact, plastic design concepts were used to determine how much structural reinforcement was necessary for the older carriers that preceded the Forrestal class. In these ships the flight deck was strengthened to accommodate heavier planes.

Probably the most significant applications of plastic theory will be in proportioning the complicated components of ship structures. Most of these components "fail" in the inelastic region. Consequently, if theories can be developed to explain this behavior in the inelastic region, improved design should result. It must be kept in mind that experience, supplemented with carefully conducted tests to failure, have been a strong influence on past design practices. In some cases, therefore, it will be found that little improvement can be made. In many other cases, substantial economies in weight, cost and design time will undoubtedly accrue.
Much of what has been learned about rolled beam sections can undoubtedly be extended to built-up sections; however, there are areas that need particular attention. Rules and procedures of design need to be established for prismatic built-up beam sections so that they perform satisfactorily in the plastic region.

Among the components that are amenable to such analysis are the following:

a) **Openings in plate structures** -- the proportioning of corner connection with cut-outs (28) and the reinforcement of deck openings.

b) **Design of deep girders** -- the problems of post buckling strength of webs, bracing requirements, and the requirements for stiffeners and their spacing. The preceding discussion concerning plate girders in bridges is also appropriate to ship girders.

c) **Stiffening requirements in flanges and corners.**

d) **Design of longitudinally stiffened plates** -- some
of the problems here are the effect of stiffener
size and spacing (minimum weight design), the
influence of lateral pressure, the influence of
residual stresses, the evaluation of boundary
conditions, and the lateral buckling of stiffeners.

Brief comment will now be made about one of the latter
problems, namely, the influence of lateral pressure on the
stability of stiffened panels.

The bottom plating of a ship is subjected to the com-
bined action of uniformly distributed lateral loading due
to water pressure and axial compression due to hogging.
The behavior of panels under these loads involves the
problem of inelastic stability. There are some experi-
mental data and methods of analysis available concerning
the stability problem of stiffened plate panels. However,
they are either limited to the plastic range or limited
to the cases when only axial compression is applied. No
experimental data or methods of analysis seem to be
available in the inelastic range when both axial compres-
sion and lateral pressure are present.
A research project on the inelastic stability of longitudinally stiffened plate panels under lateral and axial loading is being conducted at Lehigh University under the sponsorship of the Department of the Navy, Bureau of Ships. The overall objective of this project is to: 1) study experimentally the problem with special emphasis on the effect of lateral pressure upon the capacity of panels, 2) develop an analytical method for the calculation of the strength of panels, and 3) suggest appropriate modifications of design rules if found necessary.

The most desirable results of design of a built-up member are achieved when the overall load-carrying capacity is not limited by the premature or local failure of an elementary part. Thus a longitudinally stiffened panel would be most efficient (more load for the same amount of material) if plate buckling does not occur prior to overall instability of the entire panel. It is also desirable to study the geometrical proportions required to bring the entire assembly to the yield stress level. Figure 26 shows, for example, the
effect of different b/t ratios on the capacity and behavior of a stiffened panel. The load-deflection curve for specimen T4 (b/t = 60) has a sharp peak at the ultimate axial load due to the local instability of the plate. On the other hand, test T5 (b/t = 41), subjected to the same lateral pressure not only develops greater relative strength but the smooth curve at $P_u$ shows improved deformation capacity that is in contrast with the sudden failure of the plate in T4.

*   *   *

This discussion of future trends covers but a portion of the considerable list of problems requiring further study. Not only must the problems be solved, but it is important that every effort be made to solve a problem that is perhaps more difficult -- translating the results of research into a form suitable for use in design.
ACKNOWLEDGMENT

A considerable number of investigators have participated in the research project on rigid framed structures at Fritz Laboratory, the results of which have made this paper possible. Rather than recognize them individually, it may be of interest to list the different countries which they represent. They are: Australia, Austria, China, Czechoslovakia, England, Germany, Greece, Hawaii, Japan, Hungary, Netherlands, Norway, Panama, Philippines, Russia, Switzerland, the United States. So it is indeed a fact that current progress in this country is the result of a world-wide effort. In numerous cases, it has been possible to keep in touch with these colleagues and continue cooperative investigations.

The staff at Fritz Laboratory is indebted to the American Institute of Steel Construction, the American Iron and Steel Institute, the Office of Naval Research, Bureau of Ships, and Bureau of Yards and Docks of the Navy Department, and the Welding Research Council for their generous support of the research at Lehigh University. Column Research Council has also assisted in an advisory capacity.
Acknowledgment is due the regional and district engineers of the American Institute of Steel Construction who have assisted in supplying information on plastically-designed structures. In this same capacity, thanks are extended to T. R. Higgins, M. H. Bell, and E. R. Estes, Jr. of the AISC. Others who supplied valuable information were W. C. Alsmeyer, D. L. Bedingfield, J. M. Crowley, C. S. Gray, D. C. Kline, T. Kusuda, N. J. Law, F. S. Merritt, C. Massonnet, F. W. Schutz, D. L. Tarlton, E. H. Weiss. Also, acknowledgment is expressed to N. Rao who prepared a preliminary survey on this topic.

Thanks are expressed to G. C. Driscoll, Jr., T. V. Galambos, Y. C. Yen, R. J. Kozo, R. Klein, and Mrs. Ruth Huber for assistance in the preparation of this paper.
FIGURE TITLES

1. Elastic design compared with plastic design. Allowable stress is the design basis for the former; plastic analysis bases the design on the ultimate load-carrying capacity of a structure (formation of mechanism).

2. Non-dimensional moment-curvature relationship for round pin and WF shape. Present design practice permits same load factor for both shapes and thus results in higher stress in pins.

3. Number of plastically designed structures in the United States and Canada as compared with progress in Europe.

4. Schedule of research and educational program for plastic analysis and design. Preparation of specifications and commentary followed completion of major research effort. Arrow denotes first plastic design in United States.

5. Table of Contents of "Commentary on Plastic Design in Steel".

6. Progress on Codes, Specifications, and structures.

7. Examples of structures erected in United States and Canada according to plastic designs.

9. Detail showing lateral support system for plastically designed rigid frame (courtesy, Wm. A. Milek).

10. U. S. Steel supply warehouse features plastic design of rigid frame with crane (courtesy, American Bridge Division).

11. Eight-story apartment building in Canada with continuous beams proportioned by plastic method (courtesy, Canadian Institute of Steel Construction).

12. Warehouse for Gates Rubber Company makes use of 6-span rigid frame (courtesy, Inland Steel Company).

13. Industrial plant makes use of tapered members in a rigid frame span of 100 feet (courtesy, F. W. Schutz, Jr.).

14. Rendering of church with plastically designed frame of inverted keystone shape (courtesy, Wm. A. Milek).

15. Principle reasons for use of plastic design by engineers and architects justify predictions.

16. Graph of functions for which plastic designs are used shows that a major portion involve personnel.

17. United States map showing states with plastically designed structures.

18. Chart of possible future applications of plastic design with indication of design problems to be solved and the associated research problems.

20. Tests on composite beams show that natural bond is relatively ineffective (B2-T1) and fewer shear connectors are needed to develop plastic moment (B3-T1) than those presently specified (B1-T1).

21. Three aspects of frame instability are shown, corresponding to the influence of three different loading conditions.

22. Elastic stability analysis of frame which is analogous to frame which fails in inelastic region. Dashed line shows design approximation.

23. Comparison between theory and tests of small rigid frames using "box" shape. Elastic theory is confirmed, failure in inelastic region involves reduction in critical load capacity, but partial base fixity results in remarkable increase in load capacity.

24. Moment-rotation curve of structural column showing notable agreement with theoretical predictions.

25. Load-deflection behavior of girders G3 and G5 suggests the possibility of applying plastic design to plate girders.

26. Load-lateral deflection curves of two stiffened panels contrasts local plate failure (T4) with overall stiffened panel failure (T5) with resultant increase in strength and deformation capacity.
ALLOWABLE STRESS DESIGN

Load $P$

Deflection

INHERENT MARGIN OF SAFETY

$\sigma_{\text{max}} = 20 \text{ ksi}$

PLASTIC DESIGN

Deflection

$P_u = F P_w$

Figure 1

LOAD FACTOR

PIN: $\frac{M_p}{M_{w}} = \frac{1.70}{0.91} = 1.87$

WF: $\frac{M_p}{M_{w}} = \frac{1.14}{0.61} = 1.87$

Figure 2
# COMMENTARY ON PLASTIC DESIGN IN STEEL

## TABLE OF CONTENTS

1. **INTRODUCTION**
   - 1.1 Structural Design
   - 1.2 Plasticity and Design -- Some Advantages and Limitations

2. **BASIC PRINCIPLES**
   - 2.1 Behavior of Material and Structural Elements
   - 2.2 Plastic Theory

3. **ANALYSIS AND DESIGN**
   - 3.1 Assumptions
   - 3.2 Statical Method of Analysis
   - 3.3 Mechanism Method of Analysis
   - 3.4 Other Methods

4. **GENERAL PROVISIONS**
   - 4.1 Introduction
   - 4.2 Types of Construction
   - 4.3 Material
   - 4.4 Structural Ductility
   - 4.5 Yield Stress Level
   - 4.6 Plastic Moment
   - 4.7 Loads
   - 4.8 Load Factors

5. **VERIFICATION OF PLASTIC THEORY**
   - 5.1 Basic Concepts
   - 5.2 Continuous Beams
   - 5.3 Frames

6. **ADDITIONAL DESIGN CONSIDERATIONS**
   - 6.1 Shear Force
   - 6.2 Local Buckling
   - 6.3 Lateral Buckling
   - 6.4 Variable Repeated Loading

7. **COMPRESSION MEMBERS**
   - 7.1 Introduction
   - 7.2 Reduction of the Plastic Moment Due to Axial Thrust
   - 7.3 Moment-Carrying Capacity of Columns
   - 7.4 Rotation Capacity
   - 7.5 The Influence of Lateral-Torsional Buckling
   - 7.6 Frame Stability

Figure 5
8. CONNECTIONS

8.1 Straight Corner Connections
8.2 Haunched Corner Connections
8.3 Tapered Haunched Connections
8.4 Curved Haunched Connections
8.5 Beam-to-Column Connections
8.6 Details with Regard to Welding
8.7 Details with Regard to Bolting

9. DEFLECTIONS

9.1 Introduction
9.2 Deflections in the Elastic Range
9.3 Deflections in the Plastic Range
9.4 Deflection at Ultimate Load
9.5 Step-by-Step Calculations
9.6 Approximate Deflection at Working Load
9.7 Rotation Requirements

Figure 5 continued
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Figure 11

Figure 12
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*Figure 15*
Figure 16

@ 73 items in sample, See 245.2 Notebook, 1/6
Figure 17
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Figure 18
Figure 21
Plastic Zone Relieved By Sidesway

Dimensions

Before Sidesway

After Sidesway

Analogous Frame

Buckling Curves For Analogous Frame

\[
\frac{P}{P_y} = \begin{cases} 
0.6 & \text{if } \frac{L}{r} \leq 0.1 \\
0.5 & \text{if } \frac{L}{r} \leq 2.5 \\
0.4 & \text{if } \frac{L}{r} \leq 5.0 \\
0.3 & \text{if } \frac{L}{r} \leq 7.5 \\
0.2 & \text{if } \frac{L}{r} \leq 10.0 \\
0.1 & \text{if } \frac{L}{r} \leq 12.0 \\
\end{cases} 
\]

Figure 22
Figure 23
Figure 24

Figure 25
Figure 26
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To: Members, Lehigh Project Subcommittee

Gentlemen:

Enclosed you will find for your information a copy of a paper entitled ON THE APPLICATION OF PLASTIC DESIGN. This paper was presented at the Second Naval Structural Mechanics Congress at Brown University in April and is being submitted for publication in the Conference Proceedings.

The first purpose of the paper was to make a survey of the extent to which plastic design has been applied in the United States. The second purpose was to indicate possible future applications of plastic design and the corresponding research problems yet to be solved.

May I take this opportunity to thank those of you who assisted in the preparation of this report.

Sincerely yours,

Lynn S. Beadle

Enc. 1

cc: Messrs: E. R. Estes, Jr.
    K. H. Koopman