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1946

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The Influence of Viscosity on Centrifugal Pump Performance

BY ARTHUR T. IPPEN

SUMMARY

THE wide use of centrifugal pumps in the oil industry demands that their performance characteristics for oil be predicted with reasonable assurance. A systematic study of performance changes with increasing viscosities had not been undertaken so far under controlled laboratory conditions. For the purpose of undertaking such a study theoretically and experimentally, the Hydraulic Laboratory of Lehigh University and the Cameron Pump Division of the Ingersoll-Rand Company cooperated in following through a comprehensive program of research at Lehigh University during 1944 and 1945. Over 200 performance tests for viscosities up to 10,000 SSU were completed on four variants of centrifugal pumps, employing a special test stand designed and built under war conditions.

The influence of viscosity-changes on the head, discharge, and input power characteristics of the pumps is demonstrated graphically and systematically for a wide range of viscosities and speeds. The usefulness of a special Reynolds number for pumps is demonstrated and test results are correlated on that basis. General conclusions are possible as to the influence of various features of design, since several pumps of different specific speeds were tested. A theoretical analysis establishes definitely the variables to be considered and places the discussion for the problem on a sound scientific basis, especially with regard to disk and ring-losses.

INTRODUCTION

The influence of viscosity on the performance of centrifugal pumps has not received up to the present, the systematic attention of the pump engineers which this problem deserves in view of the ever increasing use of the centrifugal pump for the transport of viscous liquids. A first pioneer effort and so far the most extensive one, was made by Professor Daugherty about twenty years ago. Various papers have appeared since then, but they were never based on more than a relatively small number of field tests carried out by various investigators with greatly differing pumps. These tests were used to derive correction curves for efficiency by extensive extrapolation of the test information on hand and must clearly be recognized as a temporary means of considering the viscosity influence.

In recognition of the need of a systematic approach to this problem under controlled conditions, the Hydraulic Laboratory of Lehigh University and the Ingersoll-Rand Company of Philadelphia, New Jersey, entered into a cooperative agreement to explore the behavior of centrifugal pumps when pumping oils. A special test stand was designed and constructed under war-time restrictions and over two hundred performance runs were made during 1944–1945 on four variants of centrifugal pumps with viscosities ranging up to 10,000 SSU. The influence of viscosity changes on the head, capacity and input-power characteristics was systematically explored for each variant of pump and for several speeds. The usefulness of the Reynolds number for the presentation of these characteristics was demonstrated convincingly and all results are correlated on that basis. The data presented in the diagrams are those obtained in the present investigation only. No attempt was made to include information from other sources, since it was almost always found lacking in completeness. This situation is analogous to that existing for many years in the field of pipe friction, until finally the pertinent variables were definitely established and a final solution found.

It is to be hoped that the experiments reported here will bring forth many detailed experimental results in complete form from the files of various investigators, so that the conclusions drawn from this work may be either confirmed or be modified to fit into a more general picture. However, it is felt that the paper represents the first attempt to analyze the test results on the basis of all pertinent variables so that individual influences can be isolated more easily. Thus their share in the overall effect of the viscosity on the performance of centrifugal pumps is to be recognized in relatively true proportions.

It may be mentioned that the paper had to be severely limited in presenting the information on hand. The descriptive part on the experiments had to be cut more than may be desirable from the viewpoint of clarity in order to include the fundamental results of greatest value to the engineer concerned with this problem. The same limitations had to be imposed on the theoretical part which covers only those aspects which are most useful and characteristic in explaining the results of the experiments as stated in graphical form.

EXPERIMENTAL PART

GENERAL OUTLINE OF TEST PROGRAM

The stated purpose of the experimental study was to find, if possible, a specific relationship between viscosity on one hand and efficiency, head, capacity, and power input on the other hand. The results for four different variants of pumps were to be correlated. The choice of pumps was naturally restricted by the power supply of the hydraulic laboratory, by the range of the torsion-dynamometer, and by the available storage space for the oils used in the tests.

(a) Specifications of Pumps Tested. Under the circumstances the four pumps listed in Table I were chosen as most suitable and as of widest possible use in the pumping of highly viscous oils. All vital dimensions and normal performance data for water are listed in Tables I and II. Pump No. 1 was a single stage, single suction pump, which could be fitted with hydraulically identical impellers of the closed and open type. This pump was relatively small and of low specific speed. The pump No. 2 was a single stage, double suction pump which could be run with impellers of different diameter. Thus the specific speed was varied to a value of almost 3000 with a small impeller, however not without some sacrifice in efficiency. The experiments covered consequently a range of specific speeds between 1000 and 3000 which is the range of most efficient operation for radial flow pumps.

1 Associate Professor of Hydraulics, Department of Civil & Sanitary Engineering, Massachusetts Institute of Technology. Assoc. Member, A.S.C.E.

Contributed by the Hydraulic Division for presentation at the Annual Meeting of The American Society of Mechanical Engineers, New York, N. Y., November 27, 1945.

Discussion on this paper will be accepted until January 31, 1946.
(b) Properties of Liquids Employed. The program of testing contemplated originally the use of water and of two oils as experimental liquids. For the latter the range of viscosities was defined by the range of temperatures seasonably obtainable, i.e., from almost normal outside temperature upward to a maximum produced by dissipating the power input. No special heating or cooling equipment was provided. The variation in the volume circulated was sufficient to obtain the desired range of temperatures and viscosities. Two Black Oils, giving a medium slope in the viscosity-temperature chart, were selected and are referred to subsequently as Heavy Oil (HO) and Light Oil (LO). A third oil was added however, after the LO-runs were taken, since it appeared desirable to bridge the gap between the viscosity of water and that of the Light Oil.

This so-called Thin Oil (TO) was produced by mixing the Light Oil with a certain amount of fuel oil. The Light Oil proved to be extremely stable and its viscosity remained constant for almost half a year of intermittent testing. The Thin Oil and the Heavy Oil were affected somewhat by the pump testing and frequent analyses were made to adjust results for changes in viscosity whenever they exceeded the limits of permissible errors. Fig. 1 presents the essential data on viscosities and specific gravities for these oils.

(c) Schedule of Testing Program. The experiments started by determining the performance of each impeller of the two pumps for water. All impellers were tested as summarized in Table II for three different speeds with the exception of the largest impeller, which was run only for the two lower speeds, since the torque for the high speed exceeded the dynamometer capacity. The water tests checked very closely the performance of the pumps reported from the Ingersoll-Rand Laboratory and served in addition the purpose of establishing working procedures and of finding out about the general performance of the test stand. After some minor alterations were completed, the four sets of runs

<table>
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<th>File No.</th>
<th>Type</th>
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<th>D4</th>
<th>No.of Vanes</th>
<th>D1</th>
<th>D2</th>
<th>D3hn</th>
<th>B2</th>
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TABLE NO. II

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<td>1240 1860</td>
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Graphic Notations

- - - O O O O
were repeated for the Light Oil, covering usually the possible range of viscosities on the same day for any one speed. It was preferred to change the pumps and impellers rather than the liquids, thus avoiding contamination of one oil by the other, which would have become appreciable during the course of the testing program. The Light Oil was then cut by the addition of fuel oil to a lower viscosity and the four sets of runs were again repeated for this so-called Thin Oil. Finally the Heavy Oil was turned into the lines and the process was repeated a fourth time.

Thus two pumps with two impellers, as described in Table I, were tested for four different liquids and for three different speeds. Six performance runs were taken for each oil in the average, so that the total number of tests exceeded 220.

**Experimental Equipment**

The experimental equipment was largely assembled under the restrictions of the war-time economy and therefore contains whatever could be made to serve on the test stand, from the material resources of the two cooperating agencies. The result of the planning on this basis, however, was adequate in every respect for the proper execution of the testing program.

(a) The general arrangement of the circulating system is shown in Fig. 2. Six 750-gallon tanks formed the basic storage units for two different oils. A triangular arrangement of three tanks each in two stories was decided to be the most compact and practical one with respect to piping and housing. When the pipes were filled with one oil, two additional tanks contained the same oil on two levels with the upper tank in the rear always empty and ready to be used as a volumetric measuring tank. The upper tanks could be drained very fast by gravity into the lower tanks. The entire system of pipes and tanks may easily be analyzed from the isometric views of Fig. 2. The meter calibration circuit and the pump testing circuit are shown separately and are outlined by heavy lines. All tanks were open, however, all pipe ends were submerged considerably below the oil level, so that entrained air never posed any problem.

All the circulating was done by the test pumps driven by means of a torsion dynamometer, as illustrated in Figs. 3 and 4. The driving arrangement of the dynamometer was a system of pulleys and a 65-horsepower induction motor, which was powered from a special transformer set with 3-phase, 60-cycle, 220-volt alternating current. Automatic voltage regulation was provided in the nearest 4400-volt main feeder line of the University, so that serious speed variations were never encountered during tests.

(b) Measuring Equipment. Pump discharges had to be metered over a wide range and three liquid meters shown in Fig. 5 were installed, therefore, in separate lines of 8-in., 6-in. and 4-in. diameter. Orifice meters were used for the 8-in. and 6-in. lines and a 4 by 2-in. Venturi happened to be available for the 4-in. line. The meter-approaches were at least twenty-two diameters in length and the valves in the approach lines were only used in wide open or completely closed position. The operation of the meters, therefore, was satisfactory at all times. All discharge regulation was done by means of valves at the end of the circuit, so that the entire system was normally under positive pressure. The orifice meter of 4.92-in. orifice diameter in the 8-in. line was used almost exclusively for the large pump and the 4 by 2-in. Venturi meter was adequate for the small pump, so that the 6 by 3.60-in. orifice was rarely used. The orifice plates were made of 1/8-in. brass plates with a square edge of 1/16-in. and a downstream bevel of 45°. The pressure connections were located one pipe-diameter upstream and one-half pipe-diameter downstream.

The temperatures of the liquids were checked at three points by means of standard thermowells and dial-type indicating thermometers. The latter were mercury actuated and furnished with temperature compensated capillary tubes. Readings to the nearest half a degree Fahrenheit were deemed sufficient. The first well was located in the suction line of the pump about 4 ft. upstream, the second was in the manifold immediately below the discharge line connection, and the third recorded the temperature of the liquid below the meters.

For visual checks, on the discharge and suction pressures, two Bourdon type gauges were included on the instrument board shown on Fig. 6; however all discharge and suction heads used in the analysis were obtained from mercury-water manometers. The mercury manometer connected to the discharge side consisted of two differential gauges connected in series, each of 6 ft. length. A range of 50 p.s.i. could thus be covered without a balancing pressure. If the low pressure end of the manometer was placed under a 25 or 50 p.s.i. counterpressure, the range was extended to 100 p.s.i. In the end the mercury gauges arranged so as to require a minimum of corrections, saved considerable time since calibrations were unnecessary. The oil in the connecting lines was prevented from entering the water-mercury gauges by means of large pots filled with oil in the upper, and water in the lower half. The oil-water level in the pots was observed by means of glass gauges and was checked and adjusted before every run. The same arrangement was used for the differential manometer connected to the liquid meters. Due to the relatively small differences in specific gravity of the oil and water, small changes of the oil-water level in the separating pots will not
affect the readings. The speed was measured with a Chrono-Tachometer and with a revolution counter electrically operated for 1/10-minute intervals. It was read for every test point and comparisons were made with a Hassler Tachometer with results well within the precision of either instrument. The tachometer and revolution counter are connected to a synchronous motor powered from a generator mounted on the pump shaft.

**Experimental Procedures**

(a) Calibrations of Measuring Equipment. All the instruments used were subjected to several calibrations during the experimental period. The mercury gauges of course do not need special calibration, except that the Bourdon gauge tester was checked against the mercury columns. All connecting lines were freed of air and particularly the discharge gauge was checked before and after each run for air, by the use of a special water-air manometer, the latter indicating easily any errors of 0.01 ft. of water column. Balance was always required within this limit and was always obtained.

The dynamometer bars were calibrated by careful static load tests and showed a constant torque per unit deflection over the entire range. Naturally the percentage of error of the reading for any run was dependent upon the torque range covered for any particular pump and speed.

The speed measurements were checked against simultaneous measurements taken with other tachometers and were found to be consistent within one-fifth of one per cent, which was within the required limits.

The temperatures indicated by the dial thermometers were verified by the use of a mercury thermometer certified by the Bureau of Standards. The readings were also checked during runs by mercury thermometers in adjacent thermowells, which had been compared to the standard thermometers. Sticking of the dial hands was prevented by lightly tapping the glass cover.

The main problem in ascertaining the fundamental quantities for the performance of the pumps was encountered with the liquid meters. While the circulating system for calibrating purposes is indicated in Fig. 2, some of the phases of the calibrating runs merit mentioning. Noticeable temperature differences in the oil circulated had to be avoided. Therefore the oil was normally discharged from the line into the upper storage tank and from here by gravity back into the lower tank. The flow was then gradually switched over to the swing spout, thus taking the oil in the upper storage tank out of circulation. As soon as steady flow conditions were established again through the meter, the flow was deflected into the measuring tank, while a constant level was maintained in the lower storage tank by admitting oil from the upper storage tank. This procedure naturally required considerable practice but worked out very satisfactorily. The results of the meter calibrations are given in Fig. 7 and are stated in a form which deviates somewhat from the orthodox way. The discharge coefficient $C_D$ for the 4 by 2-in. Venturi meter is
plotted against the Reynolds number for the 4-in. diameter pipe divided by the discharge coefficient $C_D$. Similarly the results for the 8 by 4.92-in. orifice meter are given in terms of a correction coefficient for viscosity $K_m$ plotted against the pipe Reynolds number for the 8-in. pipe divided by the discharge coefficient. This latter quantity can be calculated directly from the differential manometer readings and the correction coefficient $K_m$ is thus obtained without trial and error. The discharge coefficient for water $C_{Dw}$ as introduced, is equal to 0.6185. The Venturi meter curve is compared to the curve given in the Fluid Meter Report of the A.S.M.E. and is seen to lie considerably below this curve. It is shown extending down to an experimental value of $C_D = 0.50$. The orifice curve is compared to Johanson's curve for a similar orifice in a one-in. diameter glass tube and shows remarkable agreement. It may even be argued that the discrepancy is due to the slight difference in the orifice-pipe diameter ratio, which was 0.615 here, while Johanson's curve holds for 0.595.
(b) Typical Performance Tests. Most operational difficulties were discovered and ironed out by running first a full series of experiments with water. Only minor improvements, however, were necessary, and definite procedures for the oil runs were formulated. The dimensionless performance curves shown in Fig. 8 are those obtained for the medium specific speed impeller IL 22 and they are typical of the range covered by individual tests. The almost complete absence of viscous effects for the normal range of operation with water was indicated by all water runs, proving if it be necessary indeed, that the Reynolds numbers normally encountered with water are in the so-called “rough flow” zone.

All performance runs with oil were carried out by recording all pertinent instrument readings photographically by means of a 35-mm. Kodak camera. Thus readings for individual points were taken speedily and simultaneously for permanent reference. Runs of ten to fifteen test points were normally completed within ten minutes, which fact considerably diminished the increase in temperature during any run. Upon termination of one run the pump was operated near normal discharge and was run continuously, until the dissipation of power had resulted in bringing the temperature to the desired higher value. The experimental procedure was then repeated. The temperature of the oil could be varied for any of the higher speeds from about 70° to 125° by continuous running within one day. Thus five to eight performance runs were obtained within a four- to eight-hour period. Shut-off conditions were usually tested by closing down fairly fast from a large discharge, thus retaining a fairly normal temperature of the oil churned in the pump. At best, however, the shut-off points are only approximate, especially for the higher viscosities.

The temperature changes in the oil could be controlled by adjusting the volume of oil in circulation. If the temperature rise was too fast the entire volume of oil could be utilized by circulating through upper and lower storage tanks. A differential temperature between upper and lower tank also permitted—to arrest the rise in temperature during any run after some practice. Small increases in temperature were permitted, say 2° to 3°F, and their influence was determined by taking additional points immediately after each run over the same range. Corrections to a constant temperature and hence to a constant viscosity could be made, therefore, in the analysis. However, usually such corrections were too small to influence the maximum efficiency in location and magnitude.
(100 - \(e\)) rather than to the efficiency \((e)\) itself and that it is kept to \(\pm 1\) per cent in the average for \((100 - e)\).

(b) Performance-Characteristics.—Complete performance characteristics were plotted for all runs taken and discrepancies were therefore easily discovered. The influence of accidental errors is thereby minimized. Curves for successive runs were compared according to the order of their viscosities. These performance curves available for every run should yield upon further analysis additional experimental information concerning relative distribution of disk and ring losses, and on the operation near shut-off as well as at capacities larger and smaller than normal. The results presented in the following therefore touch only upon one phase of the problem under discussion, since maximum efficiency and normal capacities, head, and inputs are the only quantities considered at present in order to limit the scope of this paper.

ANALYTICAL PART

THEORY OF VISCOS LOSSES IN CENTRIFUGAL PUMPS

(a) Hydraulic Losses. The so-called “hydraulic losses” in centrifugal pumps have been rather loosely defined in the past, since the thinking was predominantly influenced by the performance of pumps with thin fluids. With more viscous fluids the term “hydraulic losses” is here more definitely applied to the so-called “through-flow” losses, which are directly the result of skin friction and eddy systems along the primary path of the fluid passing through the pump. It is of course not possible to analyze these losses separately, since they depend on a considerable number of geometric variables, which may be briefly stated as follows:

1. Suction-pipe dimensions and shape of inlet which determine the state of flow at the entrance.
2. Design of pump inlet, eye diameter or its equivalent.
3. Shape, length, curvatures of impeller passages, contraction or expansion of cross sections.
4. Dimensions of volute, design of spiral and diffuser.

It follows that in general the flow is non-uniform and that therefore pipe-friction factors are not very useful here except in a very general way. In addition the curvatures encountered make any approach on the basis of boundary layer theory impossible. A few general statements may be in order, however, in view of the conditions stated.

1. Every pump follows its own law of hydraulic resistance. Only the total losses can be determined experimentally, since they are interdependent.
2. Theories of individual losses will at best give only a very approximate quantitative explanation of the pump behavior. This may be extremely valuable, however, for the designer.
3. The relative weight of the losses will shift as a function of the specific speed and of Reynolds number. While skin-friction losses similar to pipe flow may preclude at low specific speeds, “body-resistance” losses will come to the fore with a change from radial to mixed and more or less axial flow for higher specific speed. In other words, since losses depend to a considerable extent on approach conditions, the character of the flow through the pump is more and more determined by the state of flow in the approach to the pump when higher specific speeds are reached. The relative length of the pump passages decreases with increasing specific speeds.

On the basis of these remarks, experimental results are indeed
approached with hesitation. In retrospect it seems remarkable that the multitude of possible losses and geometric dissimilarities nevertheless produces statistical averages with a relatively narrow zone of deviations, as long as ring and disk friction losses remain comparable for the various pumps.

Energy losses for flows of a complex nature are customarily expressed in terms of velocity head of the mean flow. Choosing relative velocity at impeller exit as a suitable one, the difference between the head produced for water and the head for oil can be written as

$$H_1 = (H_w - H_o) = C_R \frac{v^2}{2g} = C_R \frac{v_o^2}{2g}$$

or dividing both sides by $H_o$

$$\frac{H_1}{H_o} = \left(1 - \frac{H_w}{H_o}\right) = C_R \left(\frac{Q_o}{H_o}\right) \frac{1}{2g} \cdot a_o^2$$

wherein $C_R$ is a function of Reynolds number, pump-design, and of roughness. Since $Q_o/H_o$ was found practically constant for a wide range of Reynolds numbers, $C_R$ can be obtained, if the water performance is known, from the plot of $H_w/H_o$ against Reynolds number.

The question of a suitable Reynolds number was naturally the subject of considerable analytic experimentation. Eventually no particular advantage was found in anyone as compared to another. However, the four fundamental forms stated below were calculated for all test points.

$$R_D = \frac{\omega \cdot r^2}{v} = \frac{\omega \cdot d_o}{2 \cdot v} = \frac{\phi \sqrt{2gH_e \cdot d_o}}{2 \cdot v} = 2620 \cdot \frac{Nd_o^2}{v^3 \cdot 10^4} \quad [2a]$$

$$R_H = \frac{\sqrt{H_e \cdot d_o}}{v} \quad [2b]$$

$$R_S = \frac{V_o \cdot d_o}{v} = \frac{283.7 \cdot Q_o}{d_o \cdot v \cdot 10^4} \quad [2c]$$

$$R_P = \frac{R_S \cdot d_o}{d_f} = \frac{283.7 \cdot Q_o \cdot b_2}{d_f \cdot v \cdot 10^4} = \frac{283.7 \cdot Q_o \cdot b_2}{b_1 \cdot d_f \cdot v \cdot 10^4} \quad [2d]$$

$R_D$ and $R_P$ differ essentially by the factor $\phi = \omega / \sqrt{2gH}$. Since $\phi$ very seldom differs greatly from unity, the reason for the small differences found in plotting the results against both is readily apparent. A small change in Reynolds number has little effect on the efficiency loss, while the differences due to pump design are more pronounced. The relatively best agreement between results for various pumps especially for low Reynolds numbers was obtained using $R_S$, in which the equivalent or actual eye-diameter of the impeller $d_f$ was introduced as the characteristic length. For practical reasons $R_D$ was chosen, since it is most readily calculated from known quantities. In addition disk and ring friction losses are most easily expressed as a function of $R_D$.

(b) Disk Friction. The problem of friction losses due to the rotation of a circular disk in a fluid has received the attention of investigators in theory and by experiments. A comprehensive view of this problem, however, has been lacking and thus the fact remained obscured, that the theoretical and experimental findings of the various investigators show considerable agreement, if the various phases of the problem are systematically related. Fig. 9 illustrates the basis on which the subsequent treatment rests. Two papers\(^7\) attack the following problems.

1. The disk rotating in an infinite space, filled with fluid.
2. The disk rotating in a housing of approximately equal diameter and with small clearances between disk and housing.

Case I. Von Karman\(^5\) dealt with problem (1) and established that the resistance mechanism depends entirely on the formation of a boundary layer adjacent to the surfaces of the disk, within which the tangential velocity of the fluid increases from zero to $\omega \cdot r$. The theoretical velocity distribution within this boundary-layer is given by Fig. 10 for the laminar case. This layer functions like the impeller of a pump. Since the pressure on the periphery of the disk is atmospheric, a radial flow $q_D$ results into the surrounding fluid at rest with an angular momentum $\omega \cdot r^2$ per unit of mass. On this basis von Karman established the basic equations summarized below for laminar and turbulent flow. It is to be noted at this point that all kinetic energy imparted to the fluid by disk action is dissipated in the surrounding space. In general the torque to be applied is defined for one side of the disk by:

$$T_D = C_D \cdot \rho \cdot \omega \cdot r^2 \cdot \frac{v^2}{2g} \quad [3]$$

(a) In the case of laminar flow, which is of particular interest here, the boundary layer thickness is known by:

$$\delta = \frac{2.58}{r} \sqrt{R_D} \quad [4]$$

and the discharge of the disk by:

$$q_D = 0.137 \cdot (2 \cdot \pi \cdot r \cdot \delta) \cdot \omega \cdot r$$

The so-called coefficient of friction becomes

$$C_D = \frac{1.84}{\sqrt{R_D}} \quad [5]$$

(b) For turbulent flow and a "smooth" disk, one for which all roughness irregularities remain submerged in the laminar sublayer, the boundary layer thickness is:

$$\delta = \frac{0.462}{r} \frac{1}{R_D^{0.70}} \quad [6]$$

on the basis of the seventh root law for the velocity distribution. The so-called friction coefficient becomes:

$$C_D = \frac{0.0728}{R_D^{0.70}} \quad [7]$$
Equations (5) and (7) are plotted as lines A and B in Fig. 11. The application to "rough" disks has not been carried through, but it is obvious that the treatment is analogous to the friction problem for flat plates and will give an expression involving the ratio \( \varepsilon/\delta \), wherein \( \varepsilon \) stands for absolute roughness. The scattering of experimental results obtained by different experimenters for large Reynolds numbers \( R_D \) is probably due to variations in \( \varepsilon/\delta \). This case, however, is of little importance for the problem here.

Case 2. The second case defined above was treated experimentally and theoretically by Schultz-Grunow\(^7\) and was essentially confirmed by the excellent experiments of Zumbusch as quoted in the same paper, the results of which are represented by line \( E \) in Fig. 11. The basic assumptions for this case should be clearly kept in mind for the later discussion of the differences between Karman's and Schultz-Grunow's results. The cylindrical wall surrounding the disk is assumed nearly equal in diameter to the disk and is short enough axially so that friction losses on the periphery may be disregarded. Thus the fluid is confined to the space between disk and housing and the angular momentum imparted by the disk is no longer lost to the surrounding fluid. Since the circular housing plates must exert a torque equal and opposite to the torque applied to the disk, it follows that the fluid confined to the space between them must revolve with one-half of the angular velocity of the disk, so that equal frictional shears may be developed on disk and circular housing plate. This fact also is confirmed by experiments and is a common assumption.

Schultz-Grunow obtained for the one-sided disk for (a) Laminar flow:

\[
\delta = \frac{2.17}{\sqrt{R_D}} \quad \text{[8]}
\]

\[
C_F = \frac{1.334}{\sqrt{R_D}} \quad \text{[9]}
\]

(b) Turbulent flow:

\[
C_F = \frac{0.0311}{R_D^{1/4}} \quad \text{[10]}
\]

These results are given as lines \( C \) and \( D \) in Fig. 11.

Case 3. A more general approach to the disk-friction problem becomes now possible as a result of the work discussed as cases (1) and (2). Using Karman's expressions, a general equation can be written for the frictional torque for a differential angular velocity \( \omega_D - \omega \). \( \omega \) now denotes the angular velocity of the confined fluid, while \( \omega_D \) represents the angular velocity of the disk. The results of this analysis may be reserved for a future paper, since space is lacking at this time. However, it is interesting to see the results of this analysis for the special case of \( \omega/\omega_D = 1/2 \). Introducing the latter ratio, the corresponding values of \( C_F \) are for

![Diagram of Velocity Distribution in Laminar Boundary-Layer Near Rotating Disk](image1)

![Diagram of General Reynolds Number Plot of Disk-Friction Coefficients](image2)
(a) Laminar Flow:
\[ C_D = \frac{1.300}{\sqrt{R_D}} \] ........................ [11]

(b) Turbulent Flow:
\[ C_D = \frac{0.0418}{R_D^{0.12}} \] ........................ [12]

with corresponding lines C and D in Fig. 11. It is seen that
Karman's equations applied properly check Schultz-Grunow's
results extremely well for laminar flow. For turbulent flow the
agreement is not as good, but equation [12] checks the experi­
mental evidence better than equation [11], since values for
numerical constant obtained experimentally increase from 0.037
to 0.040 with higher Reynolds numbers. The experimental result
permits another interesting conclusion, namely: the influence of
friiction on the cylindrical housing is increasing relative to the
circular wall friction, since the boundary-layer thickness \( \delta \) de­
creases with increasing Reynolds numbers, while the clearance
between disk and housing, \( S_D \), is constant. In other words, \( \omega \)
decrees in terms of \( \omega_D \).

The general conclusions for the influence of disk friction on
pump performance especially for viscous fluids are therefore as
follows. Theoretically the case discussed by von Karman (see
Case 1) represents the maximum disk friction which requires
\( \omega = 0 \) or \( \omega = \omega_D \). Neither condition exists normally in pumps.
The conditions generally met in pumps are discussed as Case 3,
with \( \omega \geq \frac{1}{\omega_D} \). The special case \( \frac{\omega}{\omega_D} = \frac{1}{2} \) gives a theoretical mini­
mum disk friction. A further, more extensive discussion must
be delayed at present.

Case 4. Special Case of Small Clearances \( S_D \). A clearance is
said to be small here, if radial velocities become zero and if the
tangential velocity gradient becomes constant across the axial
clearance \( S_D \). An elementary analysis gives
\[ C_D = \frac{x}{R_D} \frac{r}{S_D} \] ........................ [13]

Comparing this expression with the equations (11) and (4) as
modified for the case of \( \frac{\omega}{\omega_D} = \frac{1}{2} \) so that
\[ \frac{\delta}{r} = \frac{3.65}{\sqrt{R_D}} \] ........................ [14]

the following relation between \( S_D \) and \( \delta \) is derived.
\[ \frac{\delta}{r} = \frac{1.51}{2r} \frac{S_D}{2r} \] ........................ [15]

This equation represents the intersections of the \( C_D \) vs. \( R_D \) lines in
Fig. 11, where the laws of resistance change from equation (11)
to equation (13). The influence of the boundary layer dis­
appears, therefore, as soon as the total boundary-layer thickness
(2e) is of the order of magnitude of the clearance \( S_D \).

The transition is naturally a gradual one. The factor 1.50 instead of
unity is explained by the velocity distribution in the boundary
layer and by the fact that the law for small clearances takes full
effect only after the gradient of the velocity has become uniform
throughout the width \( S_D \). Necessary minimum clearances for
pumps lifting viscous liquids can be calculated from (14) and (15)
to avoid excessive disk friction. Of course the length \( \delta \) need not
enter at all, once its physical significance is recognized and one
may write directly the expression for the minimum clearance \( S_D \)
in terms of a limiting Reynolds number \( R_{DI} \):
\[ R_{DI} = 5.89 \left( \frac{d_1}{S_D} \right)^2 \] ........................ [16]

below which the disk friction would be unnecessarily large.
While (15) and (16) have been obtained on the basis of \( \omega/\omega_D = \frac{1}{2} \),
they do not necessarily depend seriously on this exact value,
since the total thickness of the boundary layers remains fairly
constant for \( \omega/\omega_D = \frac{1}{2} \). A decrease of \( \delta \) near the rotating disk will
be compensated by an increase of \( \delta \) near the circular housing
plate and vice versa.

(c) Ring Losses. Considering losses due to the wearing rings
it is clear that the discussion must be concerned with two different
types of losses:

1. Leakage of fluid through the annular space due to the
pressure difference between entrance and exit section.

2. Torque losses due to the tangential shear developed by the
rotation of the pump ring within the stationary housing ring.

The attention of the pump designer has been focused mainly
on the leakage losses, since thin liquids will readily pass in
quantity even through small clearances and since the torque
losses for such liquids remain small. It is clear that the general
thinking must be revised as soon as the pumps are used to lift
viscous fluids, which are 10 to 2000 times more viscous than
water.

After a careful scrutiny of all information on hand, 9, 10, 11, 12,
considerable analytical work, which cannot be included here,
established the following points:

(a) Leakage is not affected by rotation, as long as turbu­
rent flow in the clearance space persists.

(b) Leakage will be affected by rotation whenever rotation
changes the state of flow from laminar to turbulent in the ring­
space.

(c) In pumping oils the leakage flow is usually laminar and is
reduced very fast to an insignificant quantity. Temperature
gradients along its path, however, prevent ready calculation of its
magnitude. Volumetric efficiency is approaching unity.

(d) The torque losses due to the tangential shear developed are
increasing considerably in magnitude and greatly affect the over­
all efficiency.

(e) Reduction in leakage and increasing torque losses influence
the running temperature of the pump and therefore the overall
efficiency.

Points (c) to (e) deserve further discussion. It is clear that re­
duction in leakage concentrates the dissipation of increasing torque
losses within a smaller and smaller volume of liquid. Consider­
able heating must therefore be expected, which is fortunate in­
deed, since otherwise the viscous torque would very soon become
excessive. As it is, the oil entering the ring space is absorbing
the torque losses, is heated up with consequent reduction in vis­
cosity, which in turn reduces the viscous shear and increases
somewhat the leakage. A considerable temperature-gradient
exists along the axial length of the ring. Naturally the problem
is a rather difficult one, since the increase in temperature depends
on the flow of heat from the ring surfaces, the exchange of heat
from the rings to the oil leaking through and on the temperat­
ur-viscosity function of the oil itself. However, some analytical
solution is possible on the basis of adiabatic flow, which furnishes
at least a rather useful qualitative explanation of the phenomena
encountered in the analysis of the pump test results. The results of
this solution show that differences in efficiency must be ob-
RING-FRICTION

given pump will accentuate the tendency towards dissimilarity of hydraulic behavior, produced by the influence of the ring losses.

Figure 12: Example of Ring Friction as a Function of Speed and of $R_D$

SUMMARY OF EXPERIMENTAL RESULTS

(a) General Discussion and Methods of Presentation. All experimental data were plotted in the customary way; i.e., performance curves showing input horsepower, head, and efficiency were plotted against discharge from complete shut-off to the maximum flow, which was always larger than the normal flow. All analysis work started from these basic curves, of which there are more than 220 covering a range of viscosities from that for water to 10,000 SSU. The analysis as carried out in the following is only concerned with the effect of viscosity on the so-called "normal performance points" of the pump, i.e., on the maximum efficiency. The influence of the viscosity on the shape of the performance curves may give considerable insight into the relative distribution of the various losses. Time at present did not permit, however, any further exploitation of these results, which must be reserved for a future date. In the following are given the methods which were followed to obtain systematically the variation of maximum efficiency, head, and input-horsepower with increasing viscosities.

1. The first approach considered the normal discharge for water as a constant also for the oil runs. Corresponding efficiencies and heads were obtained from the oil-performance curves and dimensionless correction curves were plotted as a function of Reynolds number. This was feasible only for lower viscosities and seemed to provide a good basis for comparison with water runs since all average velocities remain the same for water and oil runs. Thus the influence of viscosity is shown immediately and clearly. Increasing viscosity causes changes in the velocity distributions everywhere in the pump, which tend to become more and more non-uniform. Thus the kinetic energy content of the flow must increase at the expense of the potential energy and, in addition, friction losses also grow rapidly for lower Reynolds numbers. The conversion of the kinetic energy to potential energy is naturally accompanied by higher losses. The size of the volute may not favor the same efficiency of conversion as for water. Furthermore, the velocity-head corrections are calculated on the basis of average velocities rather than on the basis of true kinetic energy contents. A large decrease in head is therefore noted together with the lowering of the efficiency. Since
the efficiency on the basis of constant $Q_0$ departs markedly from the maximum or peak efficiency for higher viscosities, the method proved very soon impractical as can be seen from Fig. 13 and was therefore given up.

2. It has been suggested sometimes, to assume a constant head, regardless of viscosity. This gives normal capacities which are very soon much lower than the one corresponding to maximum efficiency as can easily be verified on Fig. 13. However, for viscosities up to 100 SSU either method may be used without introducing large errors.

3. It was found that if the ratio $(Q_0/\sqrt{H_0})$ was assumed to remain constant as calculated from the water performance, a parabola could be plotted, which would intersect the head-discharge curves at the point of maximum efficiency, as shown in Fig. 13. This proved to be especially helpful in view of the fact that for lower viscosities it is impossible to decide by inspection only on a point of maximum efficiency and corresponding normal discharge, head and power. This method was used, therefore, in plotting the information presented in the following graphs, which represent the essential results of the study for easy reference. These graphs give the complete Reynolds number characteristics for the practical range of operation for all four pump variants.

b. Influence of Viscosity on Performance.

1. Normal Head and Capacity. A few general remarks are in order to explain Fig. 14 and 15. All heads and horsepower inputs are given in terms of their corresponding water equivalents and are plotted against $R_D$. The efficiency loss $(100 - \eta)/100$ was plotted in preference to the efficiency, since logarithmic scales are used, so that accidental errors are shown in correct proportion. All curves show the water performance points on the right-hand side in black with special marks to identify the different speeds. The points for Thin Oil are white, followed by Light Oil points in black, and finally the points for Heavy Oil are white again. This identification by liquids proved desirable, since differences in performance were discovered as the liquid was changed, even though the Reynolds numbers remained of the same order of magnitude. The explanation for these differences is given later with the reasoning relative to the shift of curves with speed. The general trend of the curves is self-explanatory with decreasing heads and with increasing power input and efficiency losses as the Reynolds number decreases. The striking influence of the disk-friction losses becomes apparent by comparing Fig. 11 with Figs. 14 and 15. The steep rise in efficiency losses is directly traceable to the change from turbulent to laminar disk-friction losses. A special curve for reduction in capacity was found unnecessary.
since the discharge \( Q_o \) can be obtained from \( H_o \) on the basis of
\[ Q_o = K \sqrt{H_o} \text{ wherein } K = Q_o/\sqrt{H_o}. \]
The latter relationship was found to hold true for Reynolds numbers \( R_D \) as low as 3000-4000.
Below these values of \( R_D \) the capacity decreased more rapidly, the head-capacity curves became very steep and the points of maximum efficiency had to be picked by inspection. This, of course, results in greater scattering of the points, since at the same time the general accuracy becomes lower. A slight shift in the capacity may produce an extremely large change in the head. However, it is felt that the lower limit of \( R_D = 4000 \)
will terminate almost any range, that might come up in actual practice.

2. Input Power. The input power curves represent a correction factor by which the water horsepower input \((\text{BHP}_w)\) as corrected for specific gravity, is to be multiplied to get the horsepower input for oil \((\text{BHP}_o)\). These curves indicate a rather large increase in the latter for Reynolds numbers, for which the head and capacity corrections are almost insignificant. This would be further proof of the contention that the decrease in efficiency and the increase in input power for Reynolds numbers near the 100,000 mark is mainly due to the disk and ring friction. It should also be noted that pumps IL 11 and IL 22 show the largest increase here, while IL 21 with a small impeller but relatively large rings, and IL 12 with an open impeller show smaller values. The rise in input horsepower for lower values of \( R_D \) is more or less linear in a log-log plot, corresponding to the change with Reynolds number of all hydraulic losses including disk and ring friction. For all pumps, differences in input power are found for constant Reynolds numbers as a function of speed. This may be explained in part by the effect of ring and stuffing-box friction as outlined in a previous section. The running temperature for higher speeds increases, this in turn causes a reduction of the shearing stresses and thereby a reduction also in the percentage of ring losses in terms of total power input. The larger the power output for a given pump, the smaller will be the ring-losses in per cent of input power for the same Reynolds number.

It is obvious then that these differences should be most conspicuous for pump IL 21 where the output was very small and that the difference between low and medium speed for pump IL 22 is much less, since the impeller of the latter was 35 per cent larger in diameter for the same inlet and ring dimensions. It is clear, furthermore, that the results for pump IL 11 will show the influence under discussion even less, since the rings here are very small in proportion to the impeller and therefore have but little effect on the total losses. The stuffing-box losses will accentuate the phenomenon, since IL 21 and IL 22 are double-suction pumps, while IL 11 is of the single-suction type. The disk-friction losses in contrast to the ring-friction losses will in general be larger for low specific speed pumps. They will therefore affect the performance of IL 11 most of all and will have a minimum effect on the behaviour of IL 21. This, of course, was one of the reasons for selecting these pumps. It should also be mentioned that the curves for power input correction and efficiency loss for the low speed are probably not of practical value, since such speeds are seldom employed. They were included in these tests mainly to bring out the effect of speed and in order to increase the range of Reynolds number for each oil.

3. Maximum Overall Efficiency. The tendencies of the head and input-power correction curves are essentially reflected in the plot of the ratios \((100 - \varepsilon)/100\) against Reynolds number. These curves will show all the discontinuities encountered with each of the other types to a somewhat larger scale. The question may be asked, why the efficiency loss was not given as a fraction of the efficiency loss for water. The argument against the latter method would be that the efficiency loss itself represents already, to a certain extent, a dimensionless factor of resistance for a given type of pump, similar to the pipe-friction factor \((f)\) for a given pipe with relative roughness \((e/d)\).

Just as it would be unwise to divide all pipe-friction factors by the special values of \((f)\) obtained for various values of \((e/d)\), so nothing could be gained by dividing all efficiency losses by the water efficiency. Since all efficiency losses remain constant with respect to Reynolds number in the range of water or air performance, while Reynolds number determines the behavior for more viscous liquids, the analogy to pipe-friction phenomena is close. The relative roughness of the pipe would find its counterpart in a specific speed modified to include a relative-roughness parameter. However, such an undertaking must be postponed until such time when more experimental material, systematically sifted and coordinated, is available. The efficiency-loss curves for low Reynolds numbers show a reversal of the trend at which the losses increase. It was pointed out before that below Reynolds numbers of \( R_D = 3000-4000 \) the capacity decreases at a much faster rate, than above those values. A special capacity correction curve should have been added, which, however, was not believed necessary in view of its negligible practical significance and of the somewhat uncertain evidence which permits a wide interpretation, as can easily be seen in Fig. 13. If wanted, it can of course be calculated from the other curves.

There is, however, an interesting explanation for the lower rate of increase of the efficiency losses, which takes into account the heat exchange in the pumps. If no heat were developed by ring, stuffing-box, and disk friction, the efficiency would reach insignificant values very fast. As it is, the heat developed when heavy oils are pumped, will slow down this development considerably. As the capacity decreases the cooling of the pump body decreases, so that the efficiency-loss curve will approach a value of unity only for very small Reynolds numbers. The ultimate performance will be such that oil can be pumped only as it is warmed up by the dissipation of almost the entire power input into heat. The fact that this tendency became apparent in the curves also points to the limits of the practical use to which centrifugal pumps may be gainfully employed in pumping viscous liquids.

Because it was desired to show the application of a different Reynolds number, the results for pumps IL 11 and IL 22 have been plotted against \( R_s = (283.7 - Q_o/d\sigma 10^{-6}) \). This, as is shown in Fig. 16, results in changing the order of magnitude of the Reynolds number by a factor of ten and since \( Q_o \) will decrease somewhat with Reynolds number, the curves will be stretched over a wider range. But essentially nothing new is gained from this plot and the difficulty of having an unknown quantity \( Q_o \) in the expression for \( R_s \) makes its usefulness quite doubtful.

GENERAL CONCLUSIONS

GENERAL REYNOLDS NUMBER CHARACTERISTICS

The results of the experimental and analytical work described so far show clearly that the resistance of a centrifugal pump can be represented by distinct curves plotted against Reynolds number \( R_D = 2920(Nd^{2/3} \nu^{-1}10^{4}) \). Three curves are normally required to describe the behavior as a function of Reynolds number \( R_D \):

1. Ratio of normal head for oil \( H_o \) to the normal head for water: \( H_o/H_w \)
2. Ratio of normal power input for oil \( \text{BHP}_o \) to the normal power input for water corrected for specific gravity: \( \text{BHP}_o/(\sigma w \cdot \text{BHP}_w) \)
3. The efficiency \( \varepsilon \) or better the efficiency loss \((100 - \varepsilon)\).

The capacity correction for the practical range of operation
$Q_w/Q_{w'}$ is equal to the square root of the head correction: $\sqrt{H_0/H_w}$.

In order to facilitate the conversion from the conventional SSU units for the viscosity to $\nu \cdot 10^5$ in square feet per second as used for the computing of Reynolds number, the equations of conversion may be stated here:

- For SSU $> 100$, $\nu \cdot 10^5 = \left(0.237 \text{ SSU} - \frac{140}{\text{ SSU}}\right) \ldots [18]$
- For SSU $< 100$, $\nu \cdot 10^5 = \left(0.244 \text{ SSU} - \frac{210}{\text{ SSU}}\right) \ldots [19]$

In Fig. 17 the Reynolds number characteristics for the "normal" pumps IL 11 and IL 22 have been summarized for the purpose of practical application. It is proposed to use these curves for making general corrections for a range of specific speeds from 800 to 2200 approximately. Values for pumps of different water efficiency may be interpolated.

It is of importance that the experimental material basic to the eventual adoption of correction curves be greatly increased by further analysis and coordination of available information and possibly by additional testing. It is naturally desirable to translate eventually all the calculations necessary at present in using these curves into graphical and tabular form for easy application in practice. It was felt, however, that it would be valuable to have first the benefit of an extensive discussion by all those interested in this problem, before too much time is spent in adapting the present results to practical use.

### Influence of Design Features on the Reynolds Number Characteristics

The general trend of the Reynolds number characteristics indicates three different zones within which the influence of the major losses is changing in weight.

1. For the range of water and air performance above $R_D =$
the efficiency losses are essentially due to the hydraulic "through-flow" losses, followed in importance by disk friction and by leakage.

2. For Reynolds numbers \( R_D \approx 10^4 \) down to \( R_D \approx 10^3 \) the increase in power input is caused mainly by rapidly growing disk and ring-friction losses, while through-flow losses increase at a comparatively low rate. The latter fact is indicated by the relatively small decrease in head and capacity, which proves that turbulent flow persists essentially throughout the pump. Leakage losses have assumed a negligible part.

3. For Reynolds numbers lower than \( R_D \approx 10^4 \) the through-flow losses increase more rapidly as indicated by a marked downward trend in head and capacity. Laminar flow conditions are gradually established for the main flow. Disk and ring losses become less dominant and due to the large dissipation of power into heat on account of the latter the general rise in the efficiency losses is retarded.

Due to the complex nature of the flow through a pump it is naturally impossible to fix any Reynolds number \( R_D \) for the beginning of laminar flow conditions for any one pump. It is even less possible to determine such a critical Reynolds number as a general criterion for all pumps. However, it is quite obvious that such a value of \( R_D \) would have little critical significance and is in no way analogous in this respect to the critical Reynolds number for pipe flow. The complexity of the losses and the non-uniform character of the flow preclude a sharp break in the line of resistance and point towards a very gradual and smooth transition between the two states of flow. This is indeed the conclusion to be drawn from the curves of Figs. 14–17.

A few additional conclusions may be drawn with respect to design characteristics: The head-correction curves show that for low viscosities and low specific speed (pump IL 11) the head

---

**Fig. 16. Results Plotted Against \( R_D \)**

**Fig. 17. General Reynolds Number Relations for Specific Speeds of 1000 to 2000**
may increase at first above that produced for water. This is believed mainly due to the large effect of disk action for such pumps, since the pumping by disk action here becomes a considerable percentage of the total capacity. Further proof for this contention is supplied by the curves for the open impeller of the same pump, which show a lower head and lower power in the same range of Reynolds number.

If ring losses are to form a small part of the total losses, abnormally low speeds should be avoided. The useful output of a given pump is increased considerably for viscous conditions of flow by increasing the speed, since the influence of ring losses is diminished. This tendency is exemplified by the curves for the higher specific speed pump IL 21, which shows the greatest improvement with higher speeds, since ring and stuffing-box losses form a large percentage of the total losses for the lower speeds.

Individual losses, which are due to "hydraulic friction," to disk action, to ring and stuffing-box friction, are not easily separated at present, but will vary apparently within reason in a proportional manner for pumps of conventional design. It may be assumed that a decrease in disk friction for a higher specific speed is compensated for in part by the relative increase in ring friction for this type of pump. However, there is still a net gain in efficiency, when compared on the basis of constant Reynolds number, and considering only the curves of Fig. 17. Compared on a percentage basis the efficiency losses differ less for low Reynolds numbers than for the water performance runs, since the curves converge rather rapidly for a certain range of $R_D$ values.

A considerable increase in the running temperature for a given pump is caused by ring and stuffing-box friction. While the evidence on hand does not permit more than a qualitative estimate of the influence of the pump temperature on the performance, it may be stated that for a given Reynolds number and a given speed the efficiency is somewhat higher and the head produced is increased, if the running temperature is high. In the range of $R_D$, where Heavy Oil (HO) and Light Oil (LO) points overlap, the curves show a distinctly higher head for the former, since its temperature is high here while the Light Oil is cold. These remarks are made at this time with the intent of calling attention to the influence of the running temperature rather than with the idea of suggesting definite answers.

Attempts have been made during the evaluation of the data to analyze the individual losses separately and to deduce from the power input the influence of disk and ring friction. While expressions for disk friction give probably the order of magnitude, the ring friction is not as easily calculated, since it is influenced to a decisive degree by large temperature changes in the ring space. For the time being, therefore, the attempt at isolation of the hydraulic losses from the input power was discontinued. But it is clear that eventually this phase of the problem will have to be solved, if further insight into the relative distribution of these losses is to be gained. This will be a very fruitful subject of further experimental studies.

**Recommendations**

On the basis of the information made available in this paper it is possible to propose some very definite future steps towards a final solution of the problem under discussion.

1. A considerable amount of detailed information, which up to date seemed irrelevant, should be disclosed by the profession as a result of the material published here. This material should be critically sifted and coordinated by a central agency on the basis of all the pertinent variables involved.

2. The information embodied in the performance curves and data, of which the results presented form an important but relatively small part, deserves further analysis. The influence of viscosity on part load and overload operation should be cleared to a certain extent. Since disk- and ring-friction losses remain more or less fixed regardless of discharge, the detailed analysis of input and output curves should yield valuable information towards separation of these losses from the hydraulic losses.

3. The problem of ring friction should be studied further analytically and experimentally. Applications towards improved stuffing-box design and use of mechanical seals may be included here.

4. Analytical studies should be undertaken to determine the influence of relative roughness and of specific speed as variables influencing the efficiency, so that eventually a set of universal performance curves may be plotted including these quantities in addition to Reynolds number.

5. Additional performance tests should be made with pumps of different design, especially with pumps of higher specific speeds. Such tests could be made on a much more economical basis now, since the pertinent variables and trends are established. Effects of changes in design should be systematically studied.

**Acknowledgment**

The study reported here represents an example of academic-industrial cooperation, which was sustained by the active interest of the administrative offices of Lehigh University and of the Ingersoll-Rand Company in this work. In this connection the author is especially indebted to Mr. W. M. Stanton of Ingersoll-Rand and to Prof. H. Sutherland of Lehigh University. Intimately connected with the technical phases of the work were Messrs. A. P. Brocklebank and H. Hornschuch, to whom the author wishes to express his particular appreciation. Mr. Ming Lung Pei was responsible for a great deal of analytical work. Thanks are due to a considerable number of other associates connected at various periods with this work and to the loyal cooperation of the staff members of the Hydraulic Laboratory of Lehigh University.

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