Closure to discussion of plastic strength of steel frames. (paper 764) supplement

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Closure to Discussion of

"PLASTIC STRENGTH OF STEEL FRAMES" (Paper 764)

by

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Mr. Benjamin has brought up a number of topics which are important but which could not be covered in a brief article designed to document the applicability of plastic analysis to structural design. Concerning joint details, the discusser correctly points out that much of the information developed could be applied in elastic design. Figs 5, 12, and 13 in the paper are cases in point, and further information with regard to such details may be found in Refs 12, 13, and in chapter 10 of Ref 14.

The author intended to leave the impression that the profession could safely adopt plastic design for certain steel structures. Of course, it will not replace all other design procedures; sometimes the design criterion will be fatigue, or buckling, or notch toughness. But in ordinary building construction such limitations are the exception. Already in England, plastic design has been used on a considerable number of structures (15) and one building was erected in Canada according to a design based on the plastic method.

Mr. Benjamin mentions the lack of information on members where a considerable length is in the plastic region. On the contrary, most of the tests have been conducted on members with one-third or more of the span under pure bending or in a condition approaching it. This is true of the examples shown in Fig 2, 3, 14b, 15, all of the "Lehigh tests" of Fig 22, and all of the tests of Fig 23. Since this "pure bending" condition is not ordinarily found in practice, the typical behavior will usually be better than that evidenced by most of these tests.
The other type of member mentioned is the tapered beam. It is true that studies of such beams have not been completed (although tests of their lateral buckling characteristics are underway at Columbia University). However, plastic design achieves its economy without the necessity for tapering the members. On the other hand, if there are instances where tapering might be desirable for other reasons, then such a beam can be analysed without difficulty by the plastic methods. This is illustrated by the following example in which a comparison is made on the basis of a uniformly-loaded, fixed-ended beam of forty-foot span. Two possible solutions will be examined; the load may be supported by a uniform member or by one tapered from the ends to midspan. In the first instance, a 30 WF 108 beam \((s = 299.2 \text{ in}^3, Z = 340 \text{ in}^3)\) on a 40-ft. span will support a total distributed load of 375 kips according to Eq. (3),

\[
\frac{P \cdot L}{8} = 2 M_p
\]

How will a tapered beam compare? In analysing the tapered member, equilibrium of moment requires that

\[
\frac{P_L}{3} = M_{p1} + M_{p2} = Z_p(WF)
\]

where \(M_{p1}\) and \(M_{p2}\) are plastic moments at the ends and center, respectively, and \(Z_p(WF)\) is the plastic modulus of the 30 WF 108 member. It is assumed that the member has a uniform taper, smaller at the center, and symmetrical about the center line. The ratio of \(M_{p1}\) and \(M_{p2}\) can be selected as any reasonable value, say \(M_{p1} = 3 M_{p2}\). Since the plastic modulus, \(E\), bears a linear relationship to \(M_p\) (Eq. 1), then \(E_1 = \frac{3}{2} E_2 = \frac{3}{2} Z_{WF}\).

For a built-up member the value of \(Z\) may be determined from

\[
Z = 2 \int_B y \, dA
\]

\[
Z = b t (d-t) + \frac{3}{4} (d-2t)^2
\]

where \(Z\) is the required plastic modulus, \(b\) the flange width, \(t\) the flange thickness, \(v\) the web thickness, and \(d\) the depth of the member. If the flange dimensions and the web thickness are selected to be about what would be found
for such members \((b = 11.0''', \ t = .75'', \ w = .625'')\), then the only unknown in Eq. (8) is the depth, \(d\), which may be obtained by solving the following quadratic (solution of Eq. (8)):

\[
\frac{W}{4} d^2 + dt(b-w) - \left(2 + \frac{t^2}{4(b-w)}\right) = 0 \quad \text{-(9)}
\]

For \(z_1 = \frac{3}{2} z_{WF} = 510 \text{ in}^3\) and \(z_2 = \frac{1}{2} z_{WF} = 170 \text{ in}^3\), the required depth at the ends is 38-in. and that at the center is 16.8-in.

Comparing the weight of the two beams, the uniform member, according to the plastic design, weighs 4320#. The weight of the plastically designed tapered beam is 4440#. Thus the two designs end up at about the same weight with a slight advantage for the uniform member. Of course there would have to be added to this the additional fabrication cost for the tapered member. Incidentally, conventional elastic design of the uniform beam would have required a section modulus of 429 in\(^3\). A 33 WF 141 section could have been used, the total weight being 5640# or an increase of 31% over the similar plastic design. In addition to showing the economy of material of plastic over elastic design, this example demonstrates that plastic design achieves economy without the necessity of tapering the members. Of course, the use of brackets is another matter and an even lighter design might have been achieved by using brackets at the ends (say for five feet) and a uniform member analyzed plastically for the remaining distance.

A discussion of load factors was particularly avoided in the paper. While studies of the maximum strength of structures will be helpful in arriving at a rational load factor, many other factors also enter into the problem. Rather than confuse the presentation of plastic behavior of structures with a discussion of the factor of safety — a discussion, incidentally, which is equally applicable to conventional elastic design — the factor of safety was selected in the paper on the basis that "ultimate load as the design criterion provides at least the same margin of reserve strength as is presently afforded in the conventional design of simple beams". Depending on the shape factor selected, this value could vary from 1.75 to 2.03; the former was chosen in the paper for illustration.
Concerning "office design costs", the techniques are relatively simple and are being learned by many practicing engineers. But the task of instruction and education belongs with the colleges and universities, and many now include the subject in their curricula.

The matter of code revisions comes up frequently. What form should a code or specification for plastic design take? A complete revision may not be necessary. In England and in Australia, specifications now permit use of the method, and this was done simply by adoption of an "enabling clause". A similar approach might be used in this country. Of course such a provision would be preceded by surveys and evaluations. Refs. 4, 8, 10, 14, and 16 are examples. Two other documents eventually are required and work is underway on each:

1. Rules of Practice (with commentary) for plastic design
2. A manual of design examples.

The former will include a justification of provisions, a documentation by the inclusion of test results and a summary of procedures. Ref 4 constituted a first step in the completion of the task of formulating such "rules of practice". It has now been taken on as a joint project of the ASCE (Committee on Plasticity Related to Design) and the Welding Research Council (Structural Steel Committee, Lehigh Project Subcommittee). The report is now being drafted.

Work on the Manual of Design Examples is currently underway in the offices of the American Institute of Steel Construction of which T. R. Higgins is Director of Engineering and Research.

Documents such as these should go far towards speeding the day when plastic design will be more frequently used by engineers.

Mr. Sobotka’s discussion unfortunately contains an error in the formulation of equilibrium equations (2), (2), (7), (8), and (9). All of the forces acting on the "free bodies" were not included in the moment calculation. Mr. Sobotka’s "method of forces" is the same as the "semi-graphical equilibrium..."
Mr. Sobotka's discussion unfortunately contains an error in the formulation of equilibrium equations (1), (2), (7), and (8). All of the forces acting on the "free bodies" were not included in the moment calculation. In general, a vertical shear force is present at section C whenever the member is tapered. Of course, if the member AB had been of uniform strength in the example, then the shear force would truly have been zero at section C even if the moments at A and B were dissimilar. In the tapered beam example cited earlier, the plastic hinge forms at the point of maximum moment (and zero shear) only because of the particular symmetrical arrangement chosen. Had the beam instead been tapered in the other direction (deeper at center than at ends) the hinge would not have formed at the center. Instead, two hinges would have formed simultaneously to left and right of center at those points where the moment capacity of the beam was just equal to the required statical moment of the member. The shear is of course not zero at those points. This simply underscores that the variation in moment capacity with distance along the beam must be considered in determining the location of plastic hinges therein.
Mr. Sobotta: "Method of forces" is the same as the "semi-graphical equilibrium method" noted on page 764-26, described in detail by Ref 5, and used to solve the fixed-ended beam problem (See Eq. (3)).

In closing, the authors would like to thank Mr. Sengar and Mr. Sobotta for the interesting points they have raised.

References


