Column strength under combined bending and thrust

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COLUMN STRENGTH UNDER COMBINED BENDING AND THRUST

By Robert L. Ketner

Several years ago, when the Lehigh Project Subcommittee of the Welding Research Council initiated the current program at Lehigh, it was aware that little progress could be made in studying the effect of welds upon the strength of continuous frames until more was known about the effect of other variables present. Because of this, each component part of the frame was first investigated as a separate and distinct unit. This paper presents the results to date of the column part of this total investigation with reference to strength.

As in the paper presented earlier this morning by Dr. Yang, this investigation is sponsored jointly by the Welding Research Council and the Navy Department with funds provided by those listed previously.

Since specifications for civil engineering structures are in general based on initial yielding, major emphasis has been given to this criterion of failure in the analytic investigation. However in the experimental program, tests are carried to collapse.

The strength, or carrying capacity, of a column is a
function of many variables. Some of these are:

1. loading condition
2. size of member
3. slenderness ratio
4. flexure axis
5. magnitude of axial load
6. end conditions, and
7. shape of cross-section

For the first of these, loading condition, the following were studied since they were the types most frequently encountered in tier building and portal frame design.

SLIDE NO. 4:

Load condition "a" is that condition where the end-moments are applied in such a manner that a configuration of double curvature results. For condition "b", the distinguishing feature is that one end is maintained fixed, (not allowed to rotate), while moment is applied to the other end. In condition "c", moments are applied in a manner such that the most severe condition is imposed, that of single curvature. Condition "d" is a special case of condition "a" where the length of "d" is twice that of "a".

For each of these loading conditions equations have been
developed for predicting the yield strength. To a more limited extent, methods for predicting collapse have been outlined.

SLIDE NO. 2:

Consider first the case of a small compression block subjected to an applied end-moment and an axial load.

Here is shown a curve where the applied end-moment, $M_o$, is plotted vs. axial load. The dashed curve is called the initial yield interaction curve since for any point on this curve the combined effects of the axial load and end-bending moment results in extreme fiber yielding. Also shown on this slide as a solid line is a collapse interaction curve.

If for example we now consider a column of length $L$ subjected to a condition "d" type of loading, the results will be slightly different than that shown for the small block.

SLIDE NO. 3:

The bending moment at any point on this column is composed of two parts; that due to the applied end-moment and that caused by the axial load multiplied by the deflection. Since yielding is being considered these effects can be superposed. For illustration, let us consider that part of the total moment diagram due to the applied end-moment to be fixed allowing the axial load to
vary. If the axial load is small, a condition similar to that shown in the first composite moment diagram will result. Increasing the value of P will cause a diagram such as the second curve. Here as-well-as in the first composite curve the maximum moment occurred at the end of the column. Increasing the axial load sufficiently will cause the moment diagram to be such as that shown in the last case. Here the point of maximum moment has moved away from the end of the column. Because of this movement, the interaction curve will not always be a straight line. This condition exists since yielding always occurs at the section of maximum moment. The longer the column, the less axial load will be required to cause this movement. Loading condition is also an influencing factor.

The next slide shows for various slenderness ratios the interaction curves for the different loading conditions.

SLIDE NO. 4:

Condition "a" (shown in the upper left corner) exhibits the greatest strength of those conditions investigated. As you see here, the interaction curve for a slenderness ratio of \( L/r = 112 \) closely approximates the straight line case previously discussed for the short compression block.

In the upper right, (loading condition "b"), there is a
slight reduction in the carrying capacity. This reduction is due to less restraint being offered by the loading condition.

Condition "d" (shown at the lower right) is next in line of reduction in carrying capacity. Likewise in restraint at the ends. The strength is appreciably reduced for values of axial load exceeding 50 kips.

The most severe case is shown by condition "c" (in the lower left hand corner). For this case, the interaction curve will never be the straight line previously described since the maximum moment will always occur at the center-line.

For another method of showing the effect of loading condition, let us look at the next slide.

SLIDE NO. 5:

Here the interaction curves for each of the loading conditions with slenderness ratios of 112 as shown in the previous slide have been combined. As shown before, the decrease in strength goes from loading condition "a" to "b", then "d" and finally "c". To give an idea of the relationship between these curves and present design specifications, the A.I.S.C. interaction curve is shown.

The general concept of interaction curves can be expanded...
to include another variable, Length. This we have called a Three-Dimensional Interaction Curve.

SLIDE NO. 6:

For an axially loaded member with no end-moments, the theoretical curve is the familiar column curve where load (or stress) is plotted vs slenderness ratio. This curve is based on an idealized structural steel. For members without axial load, the curves in the moment vs length plane are beam curves. The interaction curves shown previously define the surface between these extremes. Condition "a" has been shown for illustration.

Equations have been developed in the paper to be published to enable one to plot these curves.

Now to show how the tests of full size members agree with the theory.

Experimentally, this investigation is unique in two ways:

1st Structural size members are being tested in the as-delivered state

2nd The testing apparatus is such that end-moments and axial load are applied independently.

SLIDE NO. 7:

End-moments are caused by exerting forces to arms rigidly
attached to the ends of the column. These forces are accurately measured by means of aluminum tube dynamometers. The axial load was provided by means of an 800,000 lb. Rhiele Testing Machine. End thrusts, shown as E in the figure, were carried by the testing frame.

Two general types of failure were observed in the present investigation. These are shown in the next two slides.

SLIDE NO. 8:

This slide shows a column which ultimately failed due to local buckling of the compression flange elements. In all cases where the predicted collapse values were reached, failure of the column was of this type.

SLIDE NO. 9:

Here failure due to lateral buckling is shown. All condition "c" loadings failed due to this mode of failure. Because this type of collapse occurred, an analytical solution to the inelastic lateral buckling problem is an important step in further research.

For the presentation of test results, the following graphical definition of yielding and collapse have been used.

SLIDE NO. 10:

Shown here is an experimentally determined moment-rotation
curve where applied end-moment is plotted against resulting end-rotations. Going up the curve, deviation from a straight line is noticed shortly after the first yield line is observed by flaking of the white-wash on the column. This value is shown as the short, heavy dashed line perpendicular to the experimental curve. Proceeding further up the curve we come to the point we have noted as the yield strength. This point is graphically determined by extending the straight line portion of the moment-rotation curve until it intersects a horizontal tangent thru the point of maximum moment attained during the test. Thru this point construct a perpendicular to the horizontal line. Where it intersects the experimental moment-rotation curve is defined as the Yield Strength. The maximum moment the column will carry is called the Collapse Load.

The next 5 slides will show the results to date of the experimental program plotted on interaction curves. These theoretical curves are based on coupon tests for determining the yield point of the members tested. Instead of showing actual values of end-moment and axial load, the curves have been made non-dimensional by plotting $P/F_y$ vs $M/M_y$. Also included on each of these slides is the A.I.S.C. formula curve.

The first of these slides shows the results of three tests.
on SWF31 columns with L/r values of 55 tested under loading condition "b".

SLIDE NO. 11:

The heavy lines shown on this slide denote the various tests carried out for this condition. For example, T-4 was first loaded axially until the value of axial load was approximately 12%. Then holding this value constant, the moment was increased to collapse. Both the experimental initial yield and collapse agree closely with the predicted values. However as you see here, T-5 both yielded and collapsed appreciably below that predicted. Present knowledge leads us to believe that this reduction is due mainly to residual stresses in the member. These residual stresses would tend to aggravate the already present tendency toward lateral buckling.

For the same loading condition with a slenderness ratio twice that of T-3, 4 and 5 let us look at the next slide.

SLIDE NO. 12:

This figure is not entirely correct since the column would theoretically fail about its weak axis at an axial load of approximately 0.7 Fy.

The ultimate failures for each of these tests was the same
as that of the previous slide, lateral buckling. T-7 and T-9 succeeded in developing the predicted initial yield values but were unable to reach that predicted for collapse. This demonstrates a phenomenon which has been observed in the experimental program. For all cases where the moment has been maximum at the end of the column (in other words – the initial yield interaction curve is a straight line), at least the predicted initial yield value has been reached.

T-10, as in the previous case where the axial load was high, failed to develop either its predicted initial yield or collapse value.

SLIDE NO. 13:

Here we see the results of a test under loading condition "d". Both yielding and collapse occurred at a higher than predicted value. Where this condition has been observed, local buckling of the flange elements has always been the reason for ultimate collapse.

SLIDE NO. 14:

This slide shows the results of tests under loading condition "c". Here you will notice that in only one case (that where the axial load was low) was the predicted yield load reached. In no case was collapse as predicted reached.
In all tests under this condition of loading, Lateral buckling has been the reason for collapse. This is primarily due to the fact that the maximum moment always occurs at the center of the column.

**SLIDE NO. 15:**

This slide shows for a different slenderness ratio the exact same thing shown on the last slide. Lateral buckling here also governed.

Even though the present investigation is still underway, we can make the following statements:

Under certain loading conditions there is a range of \( L/r \) and \( P/F_y \) where the column will carry more than the predicted values. For this range the equations derived are safe. On the other hand, the reverse is also true. There are loading conditions where experimentally the results are considerably less than predicted. For these cases, the equations are unsafe. Further work is needed to include other variables such as residual stress, and inelastic lateral buckling. Then perhaps more consistent agreement will be found between experimental and analytical results.

Thank you.
FIG. INFLUENCE OF SLENDERNESS RATIO ON INITIAL YIELD INTERACTION CURVE
FIG. 3. LOADING CONDITION "d" THREE DIMENSIONAL INTERACTION CURVE
Fig.

END MOMENT

END ROTATION

1st Yield Line
(Determined from White Wash)

Yield Strength

Collapse
Fig.
Fig.
**Fig.**

- **T-11**
- **T-8**
- **T-12**
- **AISC**
- **JÉZÉK (SHAPE FACTOR=1.3)**
- **SIMPLE PLASTIC THEORY (L/f = 0)**
- **INITIAL YIELD**

- **8 WF 31**
- **L/f = 55**
Fig.
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<td>$\frac{L}{T} = 41$ &quot; &quot;</td>
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**Discussion: Slides**

- **B** — Stress-strain Curve - Compression Block
- **C** — Column Curve - (Comp. Block) with T-11, 15, & 18