Connections for welded continuous portal frames-
part 3, Progress report No.4
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1951

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Progress Report No. 4

CONNECTIONS FOR WELDED CONTINUOUS Portal Frames

(Part III)

Section IX: Discussion of Test Results

Section X: Summary and Conclusions

by

Lynn S. Beedle, A. A. Topractologlou, Bruce G. Johnston

This work has been carried out as a part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Institute of Research, Lehigh University
Column Research Council (Advisory)
Office of Naval Research (Contract No. 39303)

Fritz Engineering Laboratory
Department of Civil Engineering and Mechanics
Lehigh University
Bethlehem, Pennsylvania

April 17, 1951
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TABLE VIII
FOREWORD TO PART III

The majority of the figures referred to were included in Part I, distributed under date of December 7, 1950, and in Part II, distributed under date of March 31, 1951. It is suggested that these two be at hand while reading the report.

The numbering system is consistent with that commenced in the dissertation and carried out in Parts I and II.

Note to co-authors: The new figures included in Part III are not arranged in proper sequence; i.e., 155 is not necessarily followed by 156. This will be taken care of in the draft sent to the committee resulting from our to-be-agreed-upon changes.

Lynn S. Boedde
IX: DISCUSSION OF TEST RESULTS

The objectives of the investigation, outlined in Section I, form the basis for the arrangement of this discussion of test results. First, the results are examined to see if the connections satisfy the requirements of elastic design. Are the assumptions of present design practice on the safe side? Secondly, it is of interest to determine whether the behavior up to collapse will allow one to make use of any new concepts of design or analysis. Of considerable interest, of course, are the plastic methods.

1. ELASTIC STRESS DISTRIBUTIONS

1a. Straight Knees

CONNECTION P

The portions of P outside the knee behaved according to the ordinary beam theory. (Fig 53 and 54). At sections close to the knee (within 2" of the vertical stiffener) the stress distributions become irregular due to stress concentrations. However at sections removed from the stiffener a distance of half the depth of the section, the stresses are in reasonably good agreement with the computed values. The theoretical lines take into account axial thrust.

* This treatment of elastic stress distribution is incomplete, and it is expected that a separate report will follow.
The stress distribution has been discussed in Part II. The tensile stresses in the exterior beam flange are transmitted to the knee by shear and thence into the exterior column flange - (See Fig. 143*). At a particular load, these stresses are larger than the shear stress at the critical section for bending.

Referring to Fig. 57 and 58 it will be noted that whereas there are local strains that exceed the values predicted, the general trend is to confirm the assumption that the moment decreases linearly from a maximum at the flange intersections to zero at the external end of the flanges.

In the case of the beam, as shown in figure 57, no correction was made for bending in the "top" flange since SR-4 gages were only mounted on one side of the flange. Noting figure 41 and the shape of the deformed top flange in the region of the knee, it is seen that measured strains are consistent with the deflected shape.

As expected, the stress appears to be a maximum at the face of the column. At higher loads the decay of stress in the flanges and stiffeners forming the knee boundary becomes non-linear, as is typical of inelastic action.

These test results indicate that, within the elastic range, there is no serious error involved in the assumption that stresses in the stiffeners fall off linearly to zero. This statement is reserved to the type 7 in which there is no diagonal stiffener.

---

*Part II.
The theoretical stress distribution shown in Figs. 53, 54, 57 and 58 take into account both bending moment and direct stress as indicated in section V. Since the lengths of members are short, additional moment due to column deflection has not been considered. Such an additional moment will be very small.

Discussion of initial yielding is reserved for the next section.

2. ELASTIC STRENGTH OF CONNECTIONS

In this section will be discussed those aspects of connection behavior concerned with initial yield strength. The evidence and influence of residual stresses and the initial formation of yield lines as revealed by mill scale will be briefly discussed.

As is often the case with structural members, a sharp, well-defined "yield point" was not observed. The control beam test (Fig. 102) showed more rapid yielding than did any of the connections. As is evident from the summary figure 51* the transition from elastic to inelastic behavior was generally very gradual.

Consequently a definition of the initial yield strength is in order. Those values shown in Table VIII, column 4c, are the moment values at which non-linear behavior was observed by eye in the moment-rotation curves. Admittedly this is an approximate method, but since the moment-rotation curves are all plotted to the same order of magnitude, the results should be of value on a comparative basis.

*Although most of the figures previously a part of Part I and II have not been reproduced here, Fig. 51 is included at the end of this report.
As indicated in Appendix G (Fig. j) an alternate method for computing initial yield strength has its basis in the offset method for determining initial yield in coupon tests when the material does not have a sharply-defined yield point. Although the material has a sharp yield point, the structure yields gradually. The offset chosen (.02%) is double that commonly used to determine the proportional limit and corresponds to an additional "plastic strain" of about 15%. The method is illustrated in Fig. j. The results using this method are tabulated also in Table VIII, Column 4a.

Another method that has been previously suggested is illustrated in Fig. 162.

This might be termed the "structurally significant yield point", since it corresponds to a point at which connection deformations would begin to affect structural behavior elsewhere in a continuous frame. The values from the connection tests are tabulated in Table VIII, Column 4d.

When yield values determined by these two methods are finally compared to theoretical values the latter should be
found using the same methods. This may be done from the theoretical M-\( \phi \) curve in pure bending (Fig. 102). For the "M-\( \phi \)" method the factor* is 1.05 and for the "structural yield" method (Fig. 162) the factor is 1.05. It is interesting to note that the two methods give identical results in the case of the control beam, or 457 in-kips.

Comparing the "M-\( \phi \)" and "structural yield" methods very similar results are obtained. Except for connections L and M they differ by no more than 9% (See Table VIII, column 4e).

"Initial yield", those values determined by eye (column 3), are always lower than the "yield" values, as they should be; and in the case of these tests constitute a good indication of initial inelastic action. There is much more scatter between observation from tests of the same type of connection and it would seem that the "structural yield" method could well be the basis of any conclusions regarding the yield strength of the connections tested, requiring no preliminary calculations and being rapid to determine from the experimental curve. These yield loads are circled on the moment-rotation curves.**

It should be pointed out that the M-\( \phi \) method is theoretically rational only if the connection is yielding throughout the whole length, enclosed by the rotation measurement device. This is seldom realized in practice.

Two additional methods for presenting experimental yield values, not original with the writers, are indicated in Fig. 164.

* Ratio: \( \frac{M_{\text{yield}}}{M_{\text{initial yield}}} = \frac{M_{\text{yield}}}{\sigma_y \cdot S} \)

** Note to co-authors: This will be done in final draft to publish, NOT this one since figures are not duplicated.
In Fig. 164(a) it is only necessary to determine the predicted initial yield point, extend the elastic curve and find the value \( \delta_y \). The experimental value is then obtained. The scheme shown in Fig. (b) involves prescribing a decrease in slope.

In discussing the initial yield of the control beam, Fig. 102, it should be noted that the beam was tested by the same method as the beams which were described in Progress Report (40).

The member slides through vertical guides. Compared with other sections tested, the 8B13 shape has "poor" lateral buckling characteristics, particularly in the inelastic range. Thus, although greased, the guides might have provided enough friction to indicate an apparent increase in load-carrying capacity. This could explain why the points lie to the left of the theoretical values in Fig. 102 in the region of initial yield.

The calculated initial yield moments \( M_h \), shown in column 5 of Table VIII, are computed from the wedge or flexure theory (as suitable) and take into account the influence of axial thrust as well as bending moment. Prior to discussing the comparison of theory with experiment for the various connections,
the subject of shear yielding and residual stress will be discussed.

A. Shear Yield

In the prior sections of this report, information on yielding of the type 7 connections due to shear force has been discussed. In section V equations have been developed to predict the moment at which such yield should commence and to compare it with flexural yielding. Connection F provides an opportunity to examine the behavior.

In section V it was computed that "shear yield" would occur at 724 in-kips (Fig. 38) assuming uniform distribution, and 640 in-kips assuming non-uniform distribution. According to the "yield" criterion (637 in-kips) plastic deformation commenced 12% below the predicted value. Based on the "Initial Yield" value (493 in-kips) this is 32% below the predicted value. Based on the non-uniform shear distribution ($M_h(τ) = 640$ in-kips) these two percentages are 0.5% and 23%.

These values are not at all inconsistent with cantilever beam tests (which include residual stress and stress-concentration effects) which may account for a reduction in initial yield value.

However, comparing the observed yield value (637) with the initial yield moment in flexure (1195), yield commenced at about 50% of the predicted flexural yield value. As is quite evident from Fig. 38 this allowed a subsequent rotation that could not be tolerated in engineering structures.

This connection is not uncommon. Following the usual steps (8) the designer would check the shear in the beam and
column, but there is nothing in the practical literature to
tell him to check the shear within the knee panel.

In section V were found expressions for the moments at
which yielding would occur in flexure and in shear. To make
sure that the shear type of failure does not occur, then
these two moments must be equal.

\[ M_h(T) = \frac{\sigma_y}{E} \frac{w d^2}{1-d/L} \]  

\[ M_h(\sigma) = \frac{\sigma_y}{E} \left( \frac{2}{sL} \right) \left( \frac{1-d/L}{1+2L} \right) \]  

From that condition,

\[ w = \frac{s}{d^2} \left( \frac{2}{sL} \right) \left( \frac{1-d/L}{1+2L} \right) \]  

Now, the term, L, has no immediate meaning in frames since in
this discussion L is the distance from the center of the knee
to the load points on the connection.

From the proportions of connection test P, (L/d is 6.0) we
find for the shapes 14WF30, 8B13, 21WF82, 6B12, 24WF110, and
8WF31 that the third term in Eq.(42) is 0.91, the second term
ranges from 1.870 to 1.888, and the average product of the
two is 1.71.

If, instead of L/d=6.0, we select as the basis for comparison
the "Iu" value for each member, then (for the beam shapes only)
the product of the last two terms in (42) is 1.82.

Taking an average value, then, of 1.76, we obtain a sugges-
ted equation,

\[ w = 1.76 \frac{s}{d^2} \]  

The results of the test of connection P were not in too
serious discrepancy with this, and it seems that it would not
be safe to attempt to improve this expression by a more "exact"
theory of yielding.

If we use this rule, we find that none of the type 7 connections fabricated of beam shapes mentioned above are "adequate" without additional stiffening.*

Although all shapes have not been studied using equation (43) it is safe to say that most rolled shapes would probably fall into the general pattern indicated. Thus, as Griffiths has recently indicated to the writers, the AISC rules (8) should be revised to require a diagonal stiffener equal in thickness to the column flange. This, of course, is a type 8B connection. If the diagonal plate is objectionable, extra web thickness may be obtained by the use of doublers (Eq. 43). Connections of this type were not tested in the program.

An examination of Fig. 39 shows that the experimental deflection curve deviates from a straight line at approximately the same moment as that of the rotation curve. This indicates that knee deformations cause the non-linear behavior at the low loads. This is of course clear from Fig. 41 as well.

Fig. 38 demonstrates no factor of safety against yielding at the working load, 20 ksi. Thus, the connection is "inadequate" from an elastic point of view.

We may conclude that sufficient diagonal stiffening was provided in the A,K,L,M, connections to prevent serious shear deformation. This is evident from Fig. 42 and 51.

We see from the photographs of the connections at failure (Figs. 73, 79) that the diagonal did not prevent all the inelastic deformation in shear. At the higher loads yield due

* It is interesting to discover that the 8813 appears to have the best "shear" characteristics of any of the 6 shapes mentioned previously.
to shear force still occurs. However, the results of this yielding are not serious.

2B. "Yield Lines" and Residual Stress

Coating the connections with whitewash reveals the flaking of mill scale which follows the formation of Leuders Lines. These are referred to as "yield lines".

Consistent with observations made on other programs yield lines were observed at rather low loads. This information has been recorded in Table VIII (column 3). In the experimental curves it has also been designated.* This yielding is usually due to a combined residual stress and stress concentration effect. For the larger built-up connections (B,N,G,H) there is little influence on the total measured rotations. However, when the length is reduced over which this measurement is made (as in the remaining connections,) then formation of the first yield line is usually associated with the commencement of non-linearity of the load-deformation curve. This also indicates that whitewash is a rather good technique for indicating the initial inelastic action.

The consequences of this are usually not serious in the case of members designed to resist flexural loads. However, when one considers column action, wherein an additional deflection or "eccentricity" aggravates buckling, the possible seriousness of residual stress becomes important.

It is interesting to note that the first yielding in connection P occurred in a region made more sensitive due to residual stresses. Welding of stiffeners A and B will tend to create tensile residuals in the web material. B will be

* Note to co-authors: Not in Part III. Only in published version.
worse than A since it is an interior weld, and there is more restraint. The web, however, probably has a residual stress pattern (due to cooling) as sketched diagrammatically on the beam. (See Progress Report No. 1). As observed in the test, yielding has occurred where the residual stresses are additive. (Fig. B).

All of the connections were tested in the as-delivered, as-welded condition. Residuals due to cooling after rolling were measured in a previous investigation described in Progress Report 1 (40). A number of the connections were built up by welding (C-15, B-2B, G-5A, H-5A, I-5A).* The results of the tests indicate that the welding introduced residual stress patterns generally similar to those formed due to cooling after rolling. (Fig. 165).

The apparent influence of residual stress on behavior of the various types will be discussed in later sections.

* Second number is connection type.
2C. Yield Strengths of Connection Types

Examining the connections as a group with respect to initial yield (elastic strength) we may conclude that for most of the connections, present theories are adequate (except for the shear yielding) by an examination of column 6A of Table VIII. This ratio is the observed "Yield" value (Col. 4d) divided by the computed initial yield moment multiplied by a factor 1.05 (col. 5A) as discussed above. All but three of these values are above 95% and many are above 1.00. This indicates that most of the connections did not "yield" until a load was reached greater than predicted.

A second important general comparison as suggested by T.R. Higgins, is an evaluation of the maximum load carried compared to the load at which the test member is computed to yield. For the pure bending of a beam this ratio ranges from 1.12 to 1.20 and for the 8B13 is 1.15. The term, for this is \( f \), the "shape factor", the ratio for all of the connections is shown in column 8A.

Excepting connection P. (whose deficiencies have already been discussed) and connection B, the ratios are all greater than 1.20 and range from 1.27 to 1.47. These connections exhibited a reserve strength beyond the elastic limit at least as great as a simply-supported beam under pure bending.

In Table VIII yielding as defined by the various criteria is compared with theory using the following designations, according to the following table:

<table>
<thead>
<tr>
<th>Column</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>6B:</td>
<td>Observed 1st yield line/calculated initial yield</td>
</tr>
<tr>
<td>6:</td>
<td>Observed initial yield* / &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>6A:</td>
<td>Observed yield strength/1.05 x calculated initial yield</td>
</tr>
</tbody>
</table>

*Deviation from straight line of \( M_0-A \) curve.
Neglecting connection P, comparing all the tests, the three criteria give the following results:

<table>
<thead>
<tr>
<th>Category</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Max. Diff.</th>
<th>Average of all readings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st. yield line (6B)</td>
<td>.81</td>
<td>.31</td>
<td>.50</td>
<td>.56</td>
</tr>
<tr>
<td>non-linear (6)</td>
<td>1.07</td>
<td>.41</td>
<td>.66</td>
<td>.71</td>
</tr>
<tr>
<td>yield strength (6A)</td>
<td>1.18</td>
<td>.87</td>
<td>.31</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Although the scatter is considerable this shows that on the average for all of the connections:

1. The first yield line was observed at about half the computed initial yield value.
2. At about 3/4 of the computed value the departure of the experimental curve from a straight line could be detected and
3. At slightly over the computed value "yield" occurred beyond which deflections of the structure and distribution of moment could begin to be affected.

(1) **Straight Knees** (Connections A, K, L, M, P)

Connection P has been discussed in a previous section.

The various yield ratios mentioned above and contained in table VIII are reproduced below for straight knees, A, K, L, M.

<table>
<thead>
<tr>
<th>Connection</th>
<th>Yield Line</th>
<th>Eye</th>
<th>&quot;Yield&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.38</td>
<td>.65</td>
<td>1.05</td>
</tr>
<tr>
<td>K</td>
<td>.58</td>
<td>.73</td>
<td>.98</td>
</tr>
<tr>
<td>L</td>
<td>.64</td>
<td>.41</td>
<td>.95</td>
</tr>
<tr>
<td>M</td>
<td>.52</td>
<td>.50</td>
<td>.97</td>
</tr>
<tr>
<td>Average</td>
<td>.53</td>
<td>.62</td>
<td>.96</td>
</tr>
</tbody>
</table>

Compared to the other connections, the square knees are all on the "low" side. A part of this is because the rotation is measured over a shorter length on these knees than on any
of the others. Consequently the results of accumulated local yielding are in evidence sooner in a moment-rotation curve.

Examining the "yield" column in the above table and noting the construction details it appears that residual stress and stress-concentrations may be a factor in the results. The amount of vertical stiffening increases in sequence A-K-L-M. Particularly in the case of M, welding the vertical stiffener near the top should introduce tensile residual stresses. Then when a bending moment is applied which causes tension in the outer flange, local yielding will occur at a lower load than that predicted.

In the experiments M "yields" at the lowest load, then L, K, and A. As is often the case with residual stress, however, the influence disappears after rotations of about twice the value of rotation at computed initial yield. (In Fig. 42 at .008 radians all the experimental points tend to cluster).

It should be noted that there were difficulties with the measurement of rotation in connections A, K, and M in the later stages. This accounts for the dotted curves. However, the same general trends may be observed from the moment-deflection curves of Fig. 46.

(2) Tapered Haunched Knees

B, C, and N form a series of relatively large tapered knees. The observed yield and initial yield ratios are higher than the average for all connections. As seen in Fig. 44 yield lines in C and N were observed at nearly the same moment. In Connection C these lines were in the haunch web (see Fig. 85A...the lines subsequently widened as shown. In the case of the horizontal line, high shear force is a factor).
These lines are probably due to welding residual stress. Although not shown, the first line in N was in the rolled section web due to local effects.

The real yielding of C and N, however, did not start until the rolled sections started to yield (the longer of the two in each case).

In the case of B, however, inelastic action commenced almost immediately after the formation of the first yield line. Because of the nature of the connection there was probably more residual stress built up due to welding. The first yielding occurred at about 41% of the predicted value, and structural yield at about 80% of that value. Together with other factors, residual stress apparently contributed to early yielding.

Connections D, E, and F exhibited "average" behavior, that is, the average initial yield and yield ratios were nearly identical with those of all connections. See Table VIII. Comparing E with D and F which had the "full-depth tapered" stiffeners, yielding occurred in the latter at slightly lower loads. This is consistent with the pattern in the straight knees.

Referring to Fig. 47, one suggestion for the improved behavior of E over D and F in the region prior to reaching the maximum is due to residual stress. Both F and D have nominally full-depth stiffeners. E has only a half-depth stiffener, the welding of which will introduce less residual stress and constitute a less severe stress concentration.

Welding of this type will set up tensile residual stresses for D and this tension on the top flange will cause...
For E, a compression will actually be introduced at the top which should improve the characteristics, and this is seen to be the case. However, as is usually the case with residual stress effects, the influence disappears after a "reasonable" amount of yielding.

As is indicated in Figs. 82 and 81B the bracket and stiffeners were obviously sufficient to prevent any large-scale yielding in the knee area, forcing it to occur in the rolled section. In Fig. 82, comparing stiffeners "A" and "B", the former seems to be more effective, since the upper panel bounded by the two sniped stiffeners contains the greater number of yield lines. This difference is not of structural significance.

(3) Curved Knees

In this group it is seen that the first yield line occurred at a proportionately lower load than any of the other connections. However, the local yielding did not seriously influence the deformation since the "yield" ratio was higher than the average. (1.13 compared to 1.06). The curved inner flange should have high compressive residual stresses as will the web adjacent to it. In the case of I, however, with thicker curved flange, the residual stresses would not be expected to be as high as in G and H.

Many of the built-up connections gave evidence of yielding due to shear forces. Examples of this may be seen particularly in Fig. 26 and 85A. Such yielding, however, was local in character.

For the curved knees it will be noted that the equivalent of the "shape factor", (column 8A of Table VIII) includes
ratios higher than the average value for all connections which is 1.30.

(4) Strength, $M_r$, at the end of the connection (or end of rolled section)

In addition to the above comparisons it is necessary also to examine the moment $M_r$ developed at the end of the rolled section, being related to the design problem of specifying where the rolled beam shall end and where the haunch shall commence. The type 2B connection, tested as model B, will be used as the basis for discussion.

For purposes of analysis suppose we have a given knee proportioned to attach to a particular beam, Fig. 155a. In Fig. 155b, is drawn the initial yield moment capacity as a function of distance from the intersection of the neutral lines. (This is done diagrammatically and one might assume that the initial yield moment could be computed at any one section neglecting the fact that there really is a slope to the member.) The curve neglects the influence of axial load.

Suppose the point of inflection of the frame (for which the connection was proportioned) did not move under the action of the loads. This is precisely the situation in the tests being reported upon here.

If the point of inflection (or load point) were at $A$, Fig. 155c, then initial yielding would be reached when $M_h = M_1$. Notice, however, that the strength of the beam at section 1-1 has not been developed. In order for the yield strength to be reached the whole connection would have to yield and strain-harden until $M_h = M_2$ at the knee.
Fig. 155

Fig. 155a

Moments $M_B$

Moments $M_P$

Fig. 155b

Moment Capacity

Fig. 155c

Fig. 155d

Fig. 155
If the point of inflection were removed to point B, Fig. 155d, then the knee would yield when \( M_h = M_3 \), yield occurring somewhere along the haunch. Unless a plastic hinge were developed at section 1-1, and strain-hardening subsequently occur, the yield strength could not be developed at section 1-1. Only when the point of inflection is at C, Fig. 155e, do we have, at the present time, any justification for assuming that the connection will "develop the elastic limit strength at the end of the rolled section".

This point of view was kept in mind when the tests were being designed. In testing the tapered haunch and curved knees every effort was made to adjust the length of arm "a" in Fig. 155a so that a "worst loading" condition were realized. The point of inflection approximately at B, in Fig. 155d was selected as this condition, since initial yielding should occur more or less uniformly along the length of the haunch. A load-point at A was ruled out since presumably a connection would not be designed as in Fig. 155a to suit this loading (Fig. 155c). Good elastic design would call for material to be proportioned according to the moment diagram.

Generally then, it was considered that if the connection developed its yield strength at section 1-1 under the adverse condition B, that it would certainly do so if the point of inflection were removed to point C or beyond.

Looking at the problem from the design point of view, the haunch would normally be proportioned in the fashion indicated above—that is, an attempt would be made to place the material in such a manner that it would be uniformly stressed.
An examination of the behavior of the built-up connections will have to be made to see how they performed in this respect.

Referring to Fig. 155b, the horizontal line represents an initial yield \( (M_y) \) moment of 435 inch kips. Since nearly all the curves have been presented on the basis of haunch moment, \( M_h \) (intersection of neutral lines of beam and column extended), a set of proportions will have to be used to evaluate the behavior. (These proportions were worked out to evaluate the corresponding function in the case of collapse and may be found by comparing columns 7 and 8 of table VIII).

In terms of haunch moment \( M_h \), (Fig. 155d),

\[
M_h = M_4
\]

\[
M_h = 435 \times \frac{L}{a}
\]

Now, since the yield criterion will be used,

\[
M_h' = 435 \times 1.05 \times \frac{L}{a}
\]

\[
M_h' = 455 \frac{L}{a}
\]

A special calculation is necessary for connection N because of the wider tension flange.

The results of this calculation are shown in column 5B the calculated haunch moment when 105% \( M_y \) occurs (theoretically) at the end of the rolled section.

Two values are shown for connections C and N. This is because these two particular connections are not symmetrical along the two legs so that initial yield will be reached, theoretically, sooner at one end of the connection than at the other.
The ratio of the theoretical moment computed in column 5B is divided into the experimental value for "yield" (column 4d) and the results tabulated in column (6C).

An adjustment of the values in column 5B is necessary for the built-up knees due to variation in material properties. Coupons were tested from the plate material used to build up the various portions of the haunched and curved knees. The values obtained in tension were,

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_y$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8B13 flanges</td>
<td>41.8</td>
</tr>
<tr>
<td>$\frac{1}{4}$&quot; plate (B,C,G,J)</td>
<td>39.1</td>
</tr>
<tr>
<td>$\frac{3}{8}$&quot; plate (H, inner flange only)</td>
<td>39.0</td>
</tr>
<tr>
<td>$\frac{1}{2}$&quot; plate (I, inner flge only)</td>
<td>35.2</td>
</tr>
</tbody>
</table>

Column 5C of table VIII shows the moment values corrected for the difference in lower yield point and column 6D the resultant ratios.

Columns 6C and 6D summarize the "yield" behavior related to conditions at the end of the rolled section. In all of the connections except C and N significant yielding occurred at lower loads than would be predicted on the basis of initial yield at the end of the rolled section. The straight knees in particular leave something to be desired in this respect. If the influence of axial load had been considered most of the ratios would have been 1.0 or more since the values in column 5B and 5C would be decreased about 10%.

This yielding would possibly cause an increase in magnitude of deflections and stresses elsewhere in the structure.
We cannot draw final conclusions without examining the total load carried by the knee as compared with the computed value at the end of the rolled section. A similar comparison was mentioned earlier and was shown in column 8A. Column 8A compares \( M_{h(\text{max})} \) with \( M_{h(\text{theo})} \) when initial yield occurs anywhere in the knee. In column 9A a similar comparison is made (\( M_{\text{h(max)}} \div M_r \) theoretical yield) in which the values in column 8 are divided by those in 5B (or 5C), except that the additional factor of 1.05% is cancelled in this case, not being rational for this comparison. A true maximum is being compared with a computed initial yield value.

The results of such computations (column 9A) indicate that although all of the connections yielded at moments less than a value equivalent to \( M_y \) at the end of the rolled section, all of them at collapse developed strengths greater than this value except connection B.

For B (Fig. 156) the capacity curve is given by (a) The load was at B so that the moment to develop \( M_r(y) \) should have been line (b). The actual line is given by (c) at the end of the test.

![Diagram of Fig. 156](image)

The behavior of connection B requires some special attention, particularly since it is a rather common type. According to these results if a knee of this type had been
designed rather closely to the moment diagram, a re-design would be necessary, extending the haunch out toward the center of the frame until the moment at the end of the rolled section was only 86% of the allowable.

It is considered, however, that two other factors influenced the behavior of this knee: residual stress and lateral support.

As will be remembered, connection B was the first of the larger built-up knees to be tested and the vertical guide system of lateral support was used (Fig. 26). Compared with Fig. 27 this is not so stiff as that used on the later curved knees. As may be seen from Fig. 166, this support was not adequate to prevent buckling at the center.
It should also be remembered that this connection type loaded under a "worst loading" condition will be in reality under a more adverse situation than the other structures. For the curved knees the rate of change of slope of the moment-capacity curves would be much greater than the type 2B. The latter is nearly a straight line. Thus when yielding does occur, it occurs all along the length. In the case of the curved knees it is only at one point. Compare Fig. 87 with 91 for an extreme example of this. The significance of this lies in the fact that buckling of the inner compression flange should occur when the flange becomes plastic. The longer the length the worse the buckling tendency.

A duplicate test is planned to study this point further. Better lateral support provided to the compression flange—equivalent to that of the curved knees—should result in improved performance, but because of the relatively long length under simultaneous yielding, too much improvement is not to be expected.

The second factor is residual stress. As was mentioned earlier, knees such as the type 2B, completely built-up by welding, contain residual stresses, possibly of the order of in magnitude of those formed when a rolled shape cools after the rolling process. In beams it has been found that these residual stresses are not serious, only contributing to earlier deformations than predicted. But for members designed to carry compressive load any "early deformation" will contribute to early instability, as was mentioned previously.

Now, the inner flange of the knee is such a compression member. As the haunch becomes deeper it receives less and
less support from the web and tension flange, so that we should properly expect the consequences of residual stress to be more serious.

If axial load were taken into account all the ratios of column 9A would be increased. Thus the ratio for B would be 0.95.

We may conclude from this discussion that under worst loading conditions the connections will eventually develop their yield strengths at the end of the rolled section, although in some cases the rotations are rather large. The type 2B connection with straight flanges only comes within the framework of this statement if due account is taken of axial force.

The values in column 5 and those in column 5B (divided by 1.05) have been plotted in the moment-rotation curves for comparison.

Influence of obtuse angle: - straight knees

"Square" knees are considered in this paper, i.e., the girder and column meet at right angles. In the event that something greater than a right angle is present at the corner, application of the same conclusions and recommendations would seem justified. (The influence of axial load on rafter design will be more pronounced than for square knees):

As seen from Fig. 170, moment M is transferred to thrust, F, at the knee. In the square type all of the thrust must be transmitted by sheer to the web. However, flange B can transmit a component of F, its magnitude increasing as the angle increases. Thus the application of the same rules for square knees to these types would appear to be on the safe side.
3. STIFFNESS OF CONNECTIONS IN THE ELASTIC AND PLASTIC RANGE

3a. General Aspects

In this section will be discussed those aspects of the behavior that relate to load-deformation characteristics including the moment-rotation and moment-deflection curves. Both the elastic and initial inelastic behavior will be discussed together.

Referring first to Fig. 51, moment at the knee has been plotted against the average unit rotation (total rotation in the knee divided by the equivalent length, typical measurement of which is shown in Fig. 9). These are all experimentally-determined curves. The solid line is the curve determined from the control-beam test (simply-supported beam under third-point loading).*

*This solid curve has been drawn without applying a factor L/s. All of the experimental curves are M_b-values, whose meaning was previously described. Since the critical section for moment in straight knees is M_p, it would not be inappropriate in such knees to multiply all the moment values by the ratio L/s. (Fig. 167)
This figure shows that all of the so-called built-up connections exhibit an average stiffness greater than that of the rolled section (8B13) in the elastic region. Only Type 2 and 8B connections are less rigid. However, it has been noted in tests of continuous beams that residual stresses and stress-concentration may cause the deflection at the theoretical initial yield moment to be as much as twice as great as that predicted by theory. (These tests involved cantilever beams with completely built-in ends). All of the "square" type are within this limitation, and from this point of view the behavior is as good as might be expected for a rolled beam possessing a similar stress concentration.

The excess rotations in these knees, however, will increase the bending moments and deflections elsewhere in a frame of which the knee is a part. Using the 8B13 section with a column height of 10' and a beam span of 24', a calculation has been made of this increase of deflection at the centerline assuming third-point loading. The experimental values from connection L were used. The deflection was found to be about 10% greater than predicted due to the increase in moment above the value predicted on the basis of complete continuity.

Actually the center of a frame such as that examined above will itself begin to yield at a lower load since rolled sections contain residual stresses and the points of load application involve stress concentrations. This is discussed in a separate report.*

*Yang, dissertation, 1951
3B. Straight Knees.

CONNECTION P.

Fig. 38 will now be referred to again, describing the behavior of connection P.

In Fig. 38 is plotted the theoretical value of rotation in the elastic range. The measured slope of the curve is $\frac{M}{.005}$, compared with the theoretical previously computed value of $\frac{M}{.00441}$ the actual rotations are greater than predicted. The rotation of the knee is measured by two level bars mounted at the points shown in the sketch. The theoretical moment-rotation curves for the 8 and 14-inch members are indicated as dotted lines. The experimental curve is the solid line.

On the basis of equivalent length and the assumption which the designer normally makes for continuous connections (see section V) the elastic curve should lie between the two dotted curves for the 8WF31 and 14WF30. According to the theory of Part II this has been computed and is shown as the dashed line of figure 38. Thus the connection "fails" initially to be adequate because of elastic flexibility. It is more flexible in the elastic range than required for continuous connection by a factor of almost two and would thus cause the stresses to be higher elsewhere on the structure even before a flexural stress of 20 ksi were reached in the flanges.

In the vicinity of $M = 600$ in-kips, the connection commences to yield rapidly but at the same time continues to carry increased bending moment. As discussed earlier, this departure from elastic behavior is due to yielding within the knee panel due to high shear force. The increase is due to the fact that
(a) additional load is required to "yield" in shear those parts of the knee removed from the intersection of neutral lines. and
(b) the flanges provide restraint by bending as shown in Fig. 41.

It should also be noted that a more rapid acceleration of yielding occurred when the predicted \( M_{h} ( ) \) was reached. This is also evident in Fig. 38.

As the beam continued to rotate there was little increase in capacity after yielding commenced in the rolled members. Even at collapse, however, nearly all the inelastic deformation was within the knee. This is clear from Fig. 41 which shows the connection at the end of the test.

**CONNECTION A (2), Fig. 42**

It is quite evident from the rotation calculations of Part II that neither the deformation due to shear nor that due to flexure may be neglected when computing knee rotations.

Comparing the theory with experiment the measured rotation for Model A at 400 in-kips = .0044 radians. The % error on the theoretical value of .00318 radians is 38%.

We noted previously that rotation measurements for connection A were unreliable in the higher ranges. If we take only the low points, say up to 160 in-kips

\[ \varphi_2 \text{ (measured)} = .00347 \text{ rad.} \]
\[ \text{error} = 9\% \]
Notice that the knee actually rotates more than the theory would predict.

Probably the main reason for this discrepancy lies in the assumption made in deriving Eq. (2) Part II, which gives the bending rotation in type 2 connections. It was assumed that although vertical stiffeners were not present, the web transmitted an equal proportion of the reaction at the re-entrant corner. Actually larger rotations occur since this vertical stiffening has been omitted.

CONNECTIONS L,M,K, TYPE 8B (Fig. 42)

The same general observation is made here. The predicted value of rotation at 400 in-kips was .00318 rad. and the measured value .0032 rad., an error of 0.9%. Connection L is the basis for the above computations. This connection was tested with the improved rotation measurement device and with the improved lateral support system. Excellent agreement in the elastic region is thus obtained for L and M.

The simplest computation of rotation, that based on equivalent length as developed in Part II gives agreement with experiment as follows:

At M = 400 in-kips the theoretical value is,

\[ \phi_A = .00282 \text{ radian.} \]

This relationship is plotted in Fig. 42.

As noted earlier, the measured value = .00321 indicating an error of about 14%. Thus insufficient stiffening material was added to prevent completely elastic rotation greater than predicted by the simple computation. It will be interesting to see how well this simple theory agrees with tests of other sizes and shapes of cross-section.

From the discussion in section V it is evident that a
calculation could be made to obtain the required thickness of material in the web so that a type 7 knee will meet the stiffness requirements.

This requirement for knees joining rolled sections of equal depth was

$$\phi_A = \phi d.$$  \hspace{1cm} (39)

But, we had, in section V,

$$d \frac{M}{E} = d \frac{M}{E} + \frac{M h}{3EI} \frac{d}{2L}.$$  \hspace{1cm} (40)

Therefore the web thickness required to realize equal rotations is given by equating (39) to (40). In addition we will neglect terms $d(1 - \frac{d}{L})$ and $(1 - \frac{d}{2L})$ which is an assumption that is on the safe side. Then,

$$d \frac{M}{EI} = \frac{M}{wd^2G} + d \frac{Md}{3EI_F}.$$  \hspace{1cm} (41)

Replacing $I_F$ by $I$, an assumption not on the safe side,

$$d \frac{M}{EI} = \frac{M}{wd^2G} + d \frac{Md}{3EI}.$$  \hspace{1cm} (41)

Then,

$$w = \frac{3EI}{2wd^2G} = \frac{3ES}{4d^2G}.$$  \hspace{1cm} (41)

and for $E = 30 \times 10^6 \text{ psi}$, $G = 11.5 \times 10^6 \text{ psi}$

$$w = 1.97 \frac{S}{d^2}.$$  \hspace{1cm} (41)

Then, to give adequate stiffness in the elastic region a type 7 connection must have a stiffness equal to $w$ as given by (41). It will be noted that this value is somewhat greater than that given by Eq. (43), $w = 1.76 S/d^2$ computed in section 2 earlier.

Two "theoretical" moment-rotation curves are drawn in Fig. 42 in the inelastic region. In one (circular) it was assumed that the moment was uniform throughout the equivalent
length of the knee. In the other it was assumed that the knee remained elastic within the knee proper as defined by curve m of Fig. 149a.* Confining our attention to the "plastic stiffness" it is evident that the first assumption provides the best agreement with experiment over the greatest range.

However, local buckling was observed to commence at a moment of about 585 in-kips, and this corresponds to the point at which the second solution (squares) deviates in an upward slope from the experimental curve. The implication is that if local buckling had not occurred, good agreement between theory and experiment might still have been obtained at higher loads.

Since line m in Fig. 149 is not straight above Wy and since there is shear yielding present in the knee, the agreement observed in the above paragraph, while gratifying, might be highly coincidental.

Probably the factors that influence the behavior above the "proportional" limit more than any other are residual stresses and stress concentrations. Theories have not presently been devised to take into account the influence of these combined factors when computing moment-deformation curves.

Deflections

Fig. 46 shows the deflection curves of the 4 straight knees (A,K,L,M). The comparison theory is that given in section V. The experimental deflections are greater than predicted by the theory used since the latter is based on "minimum requirements" that do not take shear into account. (This deflection computation * Part II.)
is based on a uniform moment over an equivalent length of knee). A more exact comparison with the deflection curve in the elastic region could undoubtedly be obtained by the use of the "exact" predicted rotation developed in Part II.

Due to stress-concentrations, residual stress, and plastic deformation due to shear force (which occurred in spite of the additional diagonal plate as evidenced by Fig. 79) the curve becomes non-linear at relatively low loads. However, the rate of increase of deflection does not become "rapid" until a load is reached that corresponds approximately to the plastic hinge moment at the end of the rolled section.

Fig. 46 also shows the theoretical curve in the strain-hardening region. Connection L was provided with the best lateral support and the agreement between theory and experiment is good.

30. Tapered Knees

A theoretical analysis of the tapered and curved knees is not included in this report. A few comments on stiffness in the elastic and plastic range of the two types may be worthwhile.

As implied earlier, the use of any of the tapered knees tested in this program as part of a frame would assure a continuity at least as great as that implied in the assumption of complete continuity in straight knees, (Fig. 51). Consequently, if in anticipation of possible weakness, the designer wishes to increase the knee strength, any of the expedients tested would be adequate for elastic design.

Both D and E are slightly stiffer in the elastic region than F. F, of course, has the sniped stiffener over the
extension to the column inner flange. The slight discrepancy is not of significance. As was mentioned before, we apparently have a demonstration of the influence of residual stress. In Fig. 47 (also borne out in 51) it is noted that E departs from linearity at a higher load than F and D. This confirms the expectation that F and D would have additional residual stresses over E.

Other comments on the inelastic behavior of D, E, and F will be found in Section 4.

A rough comparison of stiffness in the elastic range indicates the sequence N - B - C - F, the connections being listed in order of decreasing stiffness, ("average unit rotation" obtained by dividing the total rotation by the equivalent length) as observed in Fig. 51. D, and E are not listed because of the difficulty with rotation measurement. N is stiffer because of the wider plate acting as "tension" flange.

If the minimum "throat" distances between the external corner and the inner flange are measured, they are found to vary as follows:

- N - 18"'
- B - 17"
- C - 16½"
- F - 17"

Agreement holds with observed stiffness except for F. This connection is so short that, proportionately, a greater length is under the higher moments.

Neglecting N and D, the stiffnesses fall within a narrow band more than twice the stiffness of the rolled section.

Although it may be coincidental with these tests there is a rough correlation based on the simplest of assumptions between
average unit rotation and minimum throat depth at the
haunch, \( d_h \) (Fig. 169). Now \( \phi = \frac{M}{EI} \)

\[
I \approx \frac{wd^3}{12} + 2(bt)(\frac{d}{2})^2
\]

If the web is neglected, \( I \approx d^2 \)

Then at constant moment \( M \),

\[
\frac{\phi_1}{\phi_2} = \frac{I_2}{I_1} = \frac{d_h^2}{d^2}
\]

where

\( \phi_1 \) = average unit rotation
of beam with depth \( d' \)

\( = 1.414d \) (Fig. 169a)

\( \phi_2 \) = average unit rotation
of beam with depth \( d_h \) (Fig. 169b)

Comparing B with the rolled section

curve,

\[
\frac{\phi_1}{\phi_2} = \frac{(17)^2}{(1.414 \times 8.063)^2} = \frac{289}{130} = 2.22
\]

Examining Fig. 51 at \( M = 270 \) in-kips

\( \phi_a = .0002 \) rad

\( \phi_b = \phi_2 / \phi_1 \times \phi_a = .00009 \) rad.

where

\( \phi_a \) = observed unit rotation of
rolled beam of depth \( d \)

\( \phi_b = \frac{\phi_2}{\phi_1} \phi_a. \)

The experimental average for B, C, and F from Fig. 51 is

\( \phi_b = .000075 \)

which constitutes a discrepancy of 17%.

More precisely, taking the web into account,

\[
\frac{\phi_1}{\phi_2} = \frac{wd^3 + bt d_h^2}{\frac{6}{wd^3} + bt \frac{d^2}{d_h^2}}
\]

\( \frac{\phi_1}{\phi_2} = 2.54 \)
At \( M = 270 \)

\[
\begin{align*}
\varphi_a &= 0.0002 \text{ (measured)} \\
\varphi_b &= 0.000079 \\
\end{align*}
\]

The agreement between experiment and theory is about 5%. Thus, as a matter of interest, the increase in stiffness over the rolled section is the same as the increase in moment of inertia of the minimum throat measured from the external corner.

Although this may be coincidental with these tests, it is of interest because the rather short knees are difficult to analyze and when it is desired to compute frame reactions and "center moments" more precisely it would be convenient if, over the length of the knee, the stiffness could be modified by a constant factor to be arrived at from the minimum throat dimensions. More precise methods are suggested by Griffiths\(^8\) where the length of haunch is longer.

3D. Curved Knees

Connection J was intended for comparison with D, E, and F. Noting the curve in Fig. 47, the elastic stiffness is almost identical. Actually, as may be seen from Fig. 51, the unit rotations for J were greater than F, and this is probably due to the shape of the knee, a greater length being involved in J, a part of which would only be slightly deeper than the rolled section itself. If the rotation measurement had been made over the same equivalent length then equivalent rotations might have been observed.

The stiffness of curved connections as measured by rotations Fig. 45 is in the sequence I - H - G. (decreasing). The deflections are in the same sequence (Fig. 49) since the
lengths of the rolled sections vary about the same as the total length of test specimen. This is not unexpected since the radius increases and the thickness of curved inner flange decreases in the same sequence. (Although not plotted, J comes between H and G in Fig. 45).

If we examine the unit rotations, Fig. 51, the sequence of decreasing stiffness is,

\[ G - (I-H) - J \]

I and H being very close together. In fact at 50% of the ultimate load the sequence changes to

\[ G - H - I - J \]

Note that the three connections proportioned approximately according to the AISC instructions have stiffnesses very nearly the same, averaging about three times that of the equivalent length of rolled section. J is of course considerably more flexible.

Making the same "throat" comparison for J and G as for the tapered knees, assuming \( I = f(d_h^2) \)

\[
d_h(J) = 17.0, \quad d_h^2(J) = 289
\]

\[
d_h(G) = 24.5, \quad d_h^2(G) = 600
\]

\[
d' = 11.4, \quad d'^2 = 130
\]

At \( M_h = 270 \) in-kips.

\[
\phi_J = \frac{0.0002 \times 130}{600} = 0.00009
\]

\[
\phi_G = \frac{0.0002 \times 130}{600} = 0.000045
\]

The measured values are

\[
\phi_J = 0.000096, \text{ an error of } 6.2\%
\]

\[
\phi_G = 0.000049, \text{ an error of } 8.9\%
\]

Thus this comparison agrees within 10% for the curved and tapered knees, resulting in an expression,
4. PLASTIC STRENGTH AND LOAD DEFORMATION CHARACTERISTICS BEYOND THE INITIAL PLASTIC RANGE

4A. Introduction

In Section II, Part I, were discussed the requirements upon connections from the point of view of plastic design. The three requirements noted were:

1. Connection of straight knees be able to transmit the full plastic moment, $M_p$.
2. Square knees should be as stiff as that of an equivalent length of rolled section and
3. the connection (straight) must have adequate "rotation capacity". (In an example a maximum requirement was 8 times the rotation at initial yield).

Item 2 has been discussed in the previous section. If deflection is not a matter of concern there is no harm in allowing somewhat greater rotations in the elastic range so long as the strength comes up to the $M_p$ value prior to the desired "rotation capacity". A special examination of the frame might be required in this case to make certain that the "last plastic hinge" possessed adequate rotation capacity to counteract the influence of large deformations at some previous connection.

On the other hand if deflection is critical in the design, as is often the case, then rotations beyond those implicitly assumed in design computations might have an adverse effect.*

*The problem of plastic design and deformation has been discussed in Progress Report No. 3 (79). This also includes approximate methods for computing continuous beam deflections.
In this section those items that relate to the transmission of moment and to rotation capacity will be discussed. This will include plastic strength, deformation beyond the maximum load (of importance in evaluating the energy to collapse a structure), rotation capacity, instability problems and lateral support.

Although it relates primarily to "elastic" design, one should also not overlook the relationship between the maximum moment carried and the computed initial yield value. For this, reference is made to the discussion in an earlier section, and to Table VIII, column 8A.

The behavior of all connections follow the same general pattern. The initial elastic range is followed by the elastic-plastic stage in which the rotations and deflections gradually become larger for equal increments of load. After yielding of variable extent has occurred, plastic instability is observed in the flanges tending to cause the knee to buckle sideways, deflecting laterally. With increasing loads the local buckling becomes more pronounced, the connection failing soon after the web starts to buckle plastically.

The data presented in Table VIII for plastic strengths furnishes information on three questions:

(a) How does the maximum moment $M_p$ compare with the plastic hinge value of the rolled section, $M_p$?

Given in column 9C, this consists of a comparison of $M_{h(4)}$ with $M_{h(a)}$. (Fig. 167).

(b) How does the maximum moment at the end of the rolled section, $M_r$, compare with the plastic hinge value at that point? "Was a plastic hinge developed at the end of the rolled section?"
In column 9, the ratio $M_r(4)$ to $M_r=487$ in-kips is formed.

(c) How does the maximum load carried compare with that at the computed initial yield? As discussed in Section 2, column 8A compares $M_h(4)$ with $M_h(b)$, the latter being the moment at which yielding should commence in the knee. A second comparison, related to this question is shown in column 9A:

$M_h(4)$ vs $M_h(c)$. *

Although the problem has not been studied for all the connections, one might well ask a fourth question: How does the maximum moment, $M_h$, compare with the plastic collapse moment expected? The problem does not arise for straight knees since it reduces to (a) or (b). In haunches, the important points are those corresponding to $M_r$ and $M_h$.

These questions are now examined in sequence:

(a): The very least one should expect from each connection is that it develop the predicted $M_p$-value. The tabulated ratios of Column 9C, $(M_h(\max) + 487)$ show that all of the connections are quite satisfactory under this comparison. The value 487 in-kips was the maximum moment obtained in the control beam test. (Fig. 102).

A theoretical comparison should be based on the predicted $M_p$-value, 517. This ratio is shown in column 9D. All the values are of course slightly reduced.

We may conclude that all of the connections will develop the predicted plastic hinge at the intersection of the neutral lines of beam and column; this is rather obvious for the built-up connections since so much additional material is added.

*The difference between column 9 and 9A is only a function of the shape factor, $f$, of the 8B13 section.
(b): This question has no special importance by virtue of the term "plastic design". In designing haunched connections it is necessary to determine where to "end" the rolled section and where to "start" the knee. If one wanted to make the very best use of material, then, he would examine the moment diagram and place the desired break so that the rolled section would be working to its maximum capacity.

Consequently, it is of interest to determine whether or not the connections will develop their maximum beam strengths, $M_r$, at the end of the knee, and this maximum strength is the plastic hinge value. Naturally if we made the lever arm long enough, this would certainly occur. But will it occur under the "worst loading" condition which was intended to cause the knee to reach the initial yield point simultaneously along the built-up portion—or as nearly so as possible. If so, it should certainly be adequate for knees with longer moment arms.

This comparison, then, of maximum moment in the rolled section divided by the observed maximum control beam moment is given in column 9 where the values in column 7 are divided by 487 in-kips. As in case (a), dividing by 517 would give slightly lower values.

We may conclude that all connections will not carry sufficient moment $M_H$ in order that the plastic strength of the rolled section, $M_r$, will be developed. Five are deficient.

Caution must be exercised in attempting to proportion haunched connections on the basis of simple plastic theory by means of a scheme in which the "break" between beam and
haunch were arrived at from the moment diagram. It indicates, further, the need for a specification of a safe M-value at the "break". Variations will be discussed within each type.

(c): A comparison with the elastic limit is of importance when evaluating the factor of safety from the point of view of elastic design.

As is evident from column 8A (column 8 divided by 5) all the connections have an adequate margin in this respect except P and B.

The ratio of maximum moment in the knee to the moment at the knee when the predicted initial yield is reached at the rolled section is given in column 9A. It may be concluded that all but three of the connections exhibit "shape factors" equivalent to those commonly observed in rolled sections. One of the three does not develop the yield strength, and the other two are not symmetrical.

Most of the results are expressed in terms of moment at the haunch, \( M_h \). It should be noted that if the moment arm is increased (the distance from the haunch to the point of inflection proportionately larger than for these tests) then the moment at the haunch measured in terms of the ability to carry the same rolled section moment, would be somewhat less. (Fig. 168).

It seems clear that increasing the moment arm would not make it any more difficult for the connection to develop rolled beam strength. Without making further tests, for arms greater than used in this program, the dependable haunch moment might be arrived at, by multiplying the given values by
Before discussing the strength developed within each connection classification (straight, haunched, curved), some of the problems of instability and lateral support, (which influence the strength in the plastic range) and of rotation capacity will be mentioned.

4B. Plastic Instability and Lateral Support.

All of the cross-sectional shapes, being those commercially available, are such that local buckling of flange or web elements does not occur in the elastic range. The same was also true of the members and complete connections themselves, the lengths being short enough so that elastic lateral buckling was not involved.

However, once the elastic limit is reached, both local and lateral buckling should follow rapidly since in the area of yielding the modulus reduces from about 30,000,000 pounds per sq. in. to a value close to zero. The fact that immediate
collapse does not occur has been attributed to the yielding process and this is the subject of a separate study at Lehigh.

It is stated and emphasized here, however, that plastic instability was involved in the collapse of every connection test, and in most cases brought about final collapse. The local buckling is a function of the geometrical proportion of cross-section. Since the 8B13 is light and since it is evident from other investigations to have rather "poor" characteristics, the choice of the light 8B13 section allows the presentation of information which will be on the safe side for other sections.

A marked reduction in stiffness occurs, of course, and necessitates the use of lateral support.

The three methods of lateral support have been described in Part I. The results of these tests, together with others, indicate that for members of a type similar to 8B13, the positive flex-bar type of support is much to be preferred over the vertical guide system. The latter is bound to introduce friction when the tendency to lateral deformation begins. Furthermore, restraint against lateral motion is only developed by elastic deformation, and once the member is allowed to deflect significantly, the buckling tendency is aggravated.

Local and lateral inelastic deformations start at very nearly the same time. For straight knees this may be seen by reference to Figs. 77 and 78.

The positive action of the flex-bar type as compared with the vertical guide system is clear from the comparison
indicated in the following table (numbers in parenthesis refer to figure numbers).

**Method of Support**

<table>
<thead>
<tr>
<th>Vertical Guides</th>
<th>Flex-Bars</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>K (74)</td>
<td>L (79)</td>
<td>The first buckled wave on K is not matched on both sides...is not symmetrical. On L the flex bars forced the formation of a symmetrical pattern, increasing rotation capacity. The situation is even more adverse on Connection A.</td>
</tr>
<tr>
<td>E (81B)</td>
<td>F (82)</td>
<td>The pattern of local buckling is more symmetrical on F.</td>
</tr>
<tr>
<td>B (166)</td>
<td>H (97)</td>
<td>G (98)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lateral support on G and H was adequate to cause collapse to occur in an S-shaped pattern. On B it was not. A similar comparison exists between E and F.</td>
</tr>
</tbody>
</table>

Noting Fig. 51, the result of improving the lateral support is to increase the rotation capacity. Compare D and E against F; A and K against L and M.

It was confirmed in this investigation that a rather small force is required initially to prevent sidewise deformation. However, when the flange elements buckle locally, aggravating the lateral buckling tendency, then this force increases rapidly. This is clear from Fig. 78 and 83. Bracing at knee would be adequate if it could carry about 10% of the total reaction and were tied to a rigid base.

Connections M and L, having about the same strengths, provide further opportunity for examination of the influence of lateral support. From Fig. 46, whereas M commences to lose the ability to carry load after a deflection of about .7", L continues to carry increased load. Since L has less web stiffening the importance of adequate lateral support is
Immediately evident.

Comparing connection F and D (Fig. 47) the latter has the most effective web stiffening, but F develops greater strength. Effective lateral support is the only explanation, particularly since the deflection patterns are identical in the elastic and early plastic region.

It is concluded that so far as plastic strength is concerned effective lateral support is more significant than variation in fabrication details. Although for the above two sets, inferior lateral support did not prevent the members from reaching the predicted load, the more positive support in each case increased carrying capacity and, in particular, the rotation capacity.

For connections of this size, bars of 1/8" x 1" tied into a rigid structure are satisfactory for providing support. It is recognized that this does not provide a basis for design. It does indicate however, that when the "opposite" end of the supporting member is tied into a rigid structure that a very modest size is adequate. Thus in Fig. 157, the vertical deflection of points A and B is to be avoided. Since positive support is provided at point C, such deflection is expected to be small.

![Diagram](image-url)
Complete sidewise motion of the purlin in Fig. 157 is to be avoided since such motion may occur without significant twist of the girder cross-section when both of its flanges are plastic.

Assuming the most effective lateral support, it is evident from the tests that such support must be provided as close to the expected point of yielding as possible. Referring to Fig. 51, compare the built-up knees as a group with the straight knees (A, K, L, M). In the latter the point of lateral support (the reentrant corner) coincides with the point of near-maximum moment and is the point at which flange yielding will first occur. Support is thus provided at the point at which it is needed the most.

In the curved knees, however, lateral support cannot be provided along the whole length of curved flange. Local buckling occurred at points remote from the point of lateral support and as a consequence collapse is relatively rapid. This is also borne out within the group of curved knees themselves, I and J being the shortest, possessing the "best" characteristics, G and H buckling more rapidly.

Thus in inelastic structures it is concluded that lateral support is most effective when it is provided at the point at which yielding would commence.

In the case of built-up connections, lateral support should be provided at the end of the haunches and also at the center of the compression flange of built-up haunches.

The Occurrence of Local Buckling

Although a considerable number of measurements were taken in the experimental program only a few curves have been presented of the local and lateral buckling.
Two series of tests give an opportunity to observe the onset of local buckling on a comparative basis. In the straight knees (A, K, L, M) the first observed local buckling was nearly identical in all tests (533 in-kips), the points are indicated in Fig. 51, corresponding to an uncorrected $M_h$ of 527 in-kips. Because of the method of construction, connections K, L, and M will yield in the column later than in the girder. Referring to Fig. 42, in connection L both buckling loads were recorded (520 and 588 in-kips respectively). After the column started to buckle locally, rotations increased very rapidly. However the rotation had exceeded the maximum coordinate on Fig. 42 before the unsymmetrical local buckling commenced at a moment of 616 in-kips (in Fig. 46 this corresponds to a deflection of about 1.3'). Shortly after this the connection collapsed.

The second set of data is from D, E, and F. From Fig. 47 it is evident that the local buckling load is very nearly the same in each case and local buckling is followed by more rapid deformations.

Comparing F and D the efficiency of the improved lateral support scheme is evident. Local buckling of D brought about a rather rapid collapse. Connection F however, continued to carry increased load through larger rotations.

The local buckling moments for the remaining connections are plotted in Fig. 51. The two points for connection N designate first buckling of the column flange and then of the beam flange.

It is interesting to observe that except for the straight knees there is a clustering of plotted points of local buckling.
at an average unit rotation of around .00033 radians per inch.

Since the onset of local buckling is followed almost immediately by larger deformations and in some cases by almost immediate collapse, instability failures become extremely important. The specification of proper geometric proportions of rolled shapes to prevent premature inelastic buckling is the subject of a current program at Lehigh.

4C. Rotation Capacity

Earlier in this report was pointed out the importance of adequate rotation capacity in the case of straight knees, the term being used to designate the rotation which will occur after the maximum is reached but without a substantial reduction in moment. From Appendix F it would appear (for straight knees) that certainly no greater factor than 8 times the rotation at initial yield would be required. To be sure, if the loading were such that in the elastic region the "center" and "connection" moments were nearly equal, then no rotation capacity would be required.

In general, none of the built-up knees have rotation capacities as great as straight knees L and M. The basic reason seems to be that lateral support cannot be provided at every point where local buckling is possible. Thus local buckling is "one-sided", introduces lateral deformation and collapse follows directly. Connection P is inadequate insofar as "rotation capacity" is concerned. The requirement of 8 times the total rotation at initial yield (Fig. 38) gives a required $\theta = .0033 \times 8 = .0264$ radians. At this rotation,

$$ M = 850^\circ k = 850 \frac{M_{\text{max}}}{1190} $$

$$ = (.71) M_{\text{max}} $$
At this rotation, in a real portal frame under third point loading, the hinge will have formed at the center and deflections will become exceedingly large - the load being well below that predicted.

Connections L and M are both adequate. It should be noted that M was tested with the vertical guide system of lateral support. Although a "collapse" curve was not determined for A and K, it was observed that the load was dropping rapidly. Thus, there is a definite indication that vertical stiffeners are desirable if the best lateral support cannot be obtained. It has been observed that flange local buckling is closely followed by web buckling, the deformed pattern being shown in Fig. 161.

![Diagram of flange and web buckling](image)

**Fig. 161**

Fig. 73 shows how the yielding and web crippling tend to occur above the inner flange extensions. Thus the effect of the vertical stiffeners is to prevent local web buckling directly over the flange, move it out away from the knee, allowing increased strength. Positive support by these flange extension stiffeners assists in maintaining the cross-sectional shape, increasing the rotation capacity.
Fig. 76, Connection L (8B) at the end of the test, shows that for this section proportion the half-depth stiffener is adequate, which is consistent with the above observations, considerable rotation of the cross-section being required before yielding penetrates to the centerline.

Rotation capacity, in the case of the curved knees, varies inversely with the radius of curvature of the inner flange (Fig. 51), and this is consistent; I, with its thicker flange and shorter length is more effectively supported laterally. G, with its long radius collapses more rapidly once yielding commences.

It is debatable whether rotation capacity is needed at all in built-up connections. Their place in so-called "plastic structures" is not clearly defined, since haunches are a product of one of the basic principles of elastic design: provide the most material where the expected bending moments are the greatest.

As mentioned in Progress Report 3, (79) wherever a structure can be efficiently designed by elastic methods (where depth of section may be varied to suit the moment diagram) there is doubtful advantage of plastic methods.

However, if the built-up connections were sufficiently stable after the maximum moment were reached, then one might make use of plastic analysis considerations to call upon the structure designed for one loading condition to carry the loads imposed by another.

Suppose that the frame of Fig. 158 were designed for load condition (a). Under a load condition (b), so long as $M_1 + M_2 > M_3 + M_4$ the originally-designed frame will carry
the new loading (assuming the knee has the necessary rotation capacity). Similarly for loading (c).

---

**Fig. 158**

Even if the knee has no rotation capacity, plastic behavior will still allow the frame designed for (a) loading to carry (c) if \( M_6 + M_5 < M_1 + M_2 \). A plastic "hinge" will form at the center line in the beam, after which the moment \( M_r \) will increase to \( M_1 \). Thus the extra material required by \( M_5 > M_1 \), and \( M_5 > M_2 \) may be saved.

All of the discussion presumes that the deflection requirements are met. The case of Fig. 168 (c) is one in which such an examination might be necessary.

4D. **Summarizing Discussion of Types Relating to Plastic Behavior** (Fig. 51, Table VIII)

D1. **Straight**

All four knees develop adequate "shape factor" as indicated by column 9A. Also (with A slightly deficient), they carry the full plastic moment, \( M_r \), at the end of the rolled section.

Connections L and M have adequate rotation capacity, the former due partially to improved lateral support, the
latter due to the stiffening which masked the influence of the vertical guide lateral support system that was rather ineffective in the case of built-up connections. For some applications the indicated rotation capacity of connections A and K might be insufficient.

Connection P, as mentioned earlier develops neither the required strength nor stiffness nor rotation capacity. This is due to the insufficient web thickness, in the later stages local buckling occurring together with the formation of a crack both of which were probably a factor in leading to collapse. The test was stopped as soon as a "downward" trend had been established.

Although the crack did not lead to serious trouble, the test of P(7) indicates a detail which should be avoided. Referring to Fig. C, p. 23 of 205C.6, the load in the column flange, tension in this case, must be transferred to the girder web. Since, in the vicinity of the snipe, only the web itself is available to transmit the proportion of this flange load, there is a concentration leading to failure. From this point of view - as well as economy, the use of the end plate as in type 83 is to be preferred, accomplishing a more even transfer of load.

It should be mentioned that the beam end was rough-finished creating additional stress concentrations. The failure, however, resulted after considerable necking.

D2. Tapered Haunches

Connections D, E, and F have adequate shape factors (col. 9A) and develop the full beam strength at the end of the rolled section (col. 9). The improved behavior of F over D (the latter possesses the greater stiffening) is
evidently due to effective lateral support. The same probably accounts for the improved rotation capacity of connection F. If such connections were to be used in lieu of straight knees then lateral support should be provided at the ends of the haunch if good rotation capacity is to be obtained.

Connections C and N, as tested under 45° loading, developed adequate plastic strength at one end of the haunch and not at the other. This is not surprising since one end is more severely loaded. If lateral support were provided at both ends of the haunch, improved behavior might be expected. Under such a loading condition, however, neither connection develops the yield strength at section a-a (Table VIII).

Connection B does not develop the full strength of the beam at the end of the haunch. Its behavior has been discussed considerably in section 2 of this report. An identical test with lateral support at 3 places (ends of haunch and reentrant corner) should indicate how important the lateral support aspect will be. It would not be surprising if the influence were small because the knee is loaded in such fashion that yielding occurs almost simultaneously through the whole length of the compression flange (Fig. 87). Thus local buckling can take place at many points. Since lateral support cannot be provided everywhere, the increase in strength might well be modest.

The behavior of B is most affected by the moment gradient. When related to the variation in strength of adjacent cross-section the "worst loading" condition on the haunch is similar to uniform moment on the control beam. Since the flanges on the latter yield simultaneously throughout a long length it
collapses rapidly after such yielding (Fig. 102). However, when the same beam has a sharp gradient as in the straight knees (A,C,L,M) it has no difficulty in carrying moments that are even greater than the maximum control beam moment (Table VIII column 7).

Although effective from an elastic design point of view, it does not appear desirable to allow a connection such as B to "yield".

D3. Curved Knees

All three connections, G,H, and I develop the yield strength of the rolled section and come close enough to developing the plastic hinge strength of the rolled section (Table VIII) as to be considered as satisfactory. Since they develop very nearly the same plastic strengths (column 7), then on the basis of these three tests, the procedures of the AISC for the design of curved knees (8) appears to have an additional merit in the plastic range. As noted earlier, rotation capacity improves with decrease in radius of curvature and increase of flange thickness. In fact, at a slight decrease in moment, Connection I satisfies the requirements mentioned earlier for rotation capacity of straight knees.

Prior to collapse connection J develops the yield strength but not the full strength of the rolled section. Thus a system of vertical stiffening is not as effective as would be obtained by following the AISC rules. Compared with the D,E,F, series its actual strength and rotation capacity are more favorable,

5. ECONOMY

The bending moment diagram for a rigid frame usually
falls off sharply from the center of the knee, the greater the knee distance the greater the reduction; thus the moment may be materially reduced by the time the ends of the knee are reached. The members coming in to a haunched knee may, as a consequence, be considerably lighter in weight than those required for a square knee. Therefore, the longer the knee distance, generally speaking, the greater the economy in the adjacent members.

As is clearly evident from Fig. 132, this increase in length is accompanied by an increase in cost of fabrication. Since the selection of the members of a frame is dependent on the moment diagrams, which vary for each span, height, and loading conditions, no general comparison can be made in this paper.

However, attention is called in Fig. 159 to the relation between knee lengths and the relative time of cutting and welding.

![Graph showing the relationship between knee lengths and time of fabrication.](image-url)
Except for connections N and J, it is evident that fabrication time (exclusive of handling) increases in almost direct proportion to the total length of the connection. This length is measured along the neutral lines from their intersection to the splice between the connection and the rolled shape.

Examining the straight knees as a group (Fig. 132) it is evident that connection A (2) is the most economical. However an additional factor involved here is the fact that special plant equipment might be required to make the 45° cut in large quantity production. Comparing K, L, and M, it is not evident from the tests that the increased cost of inserting vertical stiffeners is warranted unless the full plastic strength is to be developed including adequate rotation capacity. If this is the case then the use of vertical stiffeners is essential. The difference in cost between L and M is a measure of scatter since L should actually involve less welding.

Examining connections D, E, F, the use of half-depth stiffeners indicates an advantage over the full depth ones. From a strength point of view there is little to separate the two, so that some real economy might be obtained here if large numbers of connections were involved. Providing F and D do not differ because of experimental scatter (Fig. 132), it is slightly more expensive to bevel the "inner" stiffener, even though this eliminates a line of welding as compared with D. Undoubtedly, however, if the snipped plates were prepared in mass production, together with the expense of "fitting" the vertical inner stiffener inside many rolled sections, the advantages of F would outweigh the small difference.
Thus on a cost basis E is to be preferred over F and F over D.*

Beyond this, additional comparisons are difficult to make, since economy of rolled section, carrying capacity of haunch, and expense of haunch fabrication must all be considered. One comparison is possible between connection B and H. Both are about the same length (Fig. 159). B is about 25% more expensive, although this ratio would be reduced by the extra cost of material for H and the expense of rolling the curved inner flange. As is seen from Fig. 51 and the various ratios of Table VIII, the load carrying capacity of H is considerably greater than that of B. A part of this is due to reasons outlined in earlier parts of this report.

In arriving at an economical design from the overall viewpoint of cost of members and of fabrication of the joint, one might suggest beginning with a trial selection of the members. In the event the incoming members are capable of carrying the moment at the knee then a simple square knee is indicated. To be compared with the above trial design is another in which the members are lighter in weight and not capable of carrying the knee moment. The points near the joint where strengthening must begin can be ascertained from the moment diagram, and thus the knee lengths are established and the joint designed. A comparison of costs may now be made which will include cost of members and cost of knee fabrication.

* It should be noted that a program investigating additional variables is underway by one of the investigators now at the University of Texas.
The introduction of curved knees and tapered haunches into a design also allows for a savings in fabricating costs due to the fact that a smaller member is to be handled, joined, fabricated. There is a savings, not only at the haunch, but also at the crown and at field splices.

The disproportionate expense of connection J (Fig. 159) does not warrant this form of construction, since the load-carrying capacity does not meet the standards of the remaining knees.

6. FURTHER RESEARCH

In the interests of shortening this paper a separate note on proposed research has been prepared.

Generally a means of improving straight knee performance is warranted if it can be done at small extra expense. Also the behavior of "I" sections should be examined due to their proportionately thicker webs.

In the built-up connections a considerable amount of analytical work can be done before proceeding with tests, although some of the latter are warranted at the present time. Requirements of \( \frac{M_h}{M_r} \) and dependable \( M_r \)-values for various knee proportions must be established.

A method of attack into the interior connection problem has also been separately prepared. Particularly in the case of straight knees, the influence of size and shape of cross-section requires examination.
X: SUMMARY AND CONCLUSIONS

The statements and conclusions listed below are restricted to the connections tested and the rolled sections examined. However, since commercially-available material was used and since customary design practice was followed wherever possible, the results should be of general application.

A: Connection Types

(1) TYPE 2 (A)*

(a) By taking into account both shear and moment the experimental moment-rotation curve can be predicted within about 10%.

(b) The type 2 connection develops adequate strength but the type 8B connections is preferable, because of hidden cost factors and improved performance in the plastic range.

(c) Although the difference was not marked, this connection takes less time to fabricate than the 8B connections.

(2) TYPE 2B (B)

(a) This knee does not quite develop the moment at the haunch corresponding to initial yield.

Unless the influence of axial load is taken into account, it does not develop the rolled section yield strength (Col. 9A). It quite evidently will not develop the full rolled section strength.

(b) When compared to the other built-up models, the inferior behavior of this connection under the Types and connection models are indicated in Table VIII, found at the end of the report.
"worst loading condition" selected is predominantly due to the fact that the whole length of compression flange is yielded simultaneously. Residual stress is a factor since it causes this yielding to occur at a lower load than predicted, aggravating the lateral buckling tendency. An additional test with improved lateral support will demonstrate the influence of this factor, although it is suspected that it will not be large.

(c) On the basis of this one test the initial yield load is the true limiting carrying capacity. An alternate is to extend the haunch a greater distance into the frame, or to increase the size of rolled section and shorten the haunch.

(d) The average unit rotation for the knee may be determined approximately by the expression

\[ \phi_h = \frac{d_1^2}{2} \frac{M_r}{EI} \]

(3) TYPE 4 (D E F)

(a) The best design of the three tested is E. Further improvement could be realized by using a sniped full-depth stiffener as an extension to the inner column flange and by supporting the connection laterally at the ends, as in F. Further economies due to additional modification might well be made, such as the replacement of the two external stiffeners by an end plate as in type 14. Fig. 5

(b) With this type (which was not tested under a worst loading condition) the connections indicate an
an adequate reserve of strength above the yield point. The rolled section strength is fully developed at the splice.

(c) Lateral support should be provided at the ends rather than at the center (as on these tests) if good rotation capacity is to be desired in the plastic range.

(4) **TYPE 5A (G H I J)**

(a) The curved knees designed according to the AISC specifications (8) (G H and I) performed in excellent fashion. Both the initial yield strength and the full plastic strength at the end of the rolled section were developed (columns 9, 9A). The reserve strength above the predicted yield load is more than adequate (column 8A).

(b) The average unit rotation is about one-third that of the rolled beam, and for the same thickness of curved inner flange may be determined within 10% by the expression given in 2(d) above.

(c) A system of vertical stiffening, such as that used in J in lieu of increased flange thickness is not to be recommended on the basis of these tests. Proportionately the connection is expensive.

(5) **TYPE (7) (P)**

(a) External to the knee stress distribution is in accordance with ordinary beam theory until the
distance from the knee is reduced to half the depth of the section.

(b) Within the knee, flange stresses decrease linearly from the rolled section to the external end. Shear stresses in the web are larger than those at the critical section for bending.

(c) Yielding due to shear force in the web occurs at about 50% of the moment corresponding to the flexural yield point. Agreement with theory was within a few percent. The subsequent additional rotation cannot be tolerated in engineering structures and means must be taken to avoid such failure. Diagonal stiffeners (type 8B) and web doubler plates are recommended. The expression \( w = 1.76 S/d^2 \) gives the required web thickness to prevent premature web yielding.

(d) As tested, for WF sections, the connection is inadequate from both elastic and inelastic strength and stiffness point of view.

(e) In the elastic range this type is more flexible than predicted by the theory developed. Further it is twice as great as that implied in ordinary deflection calculations on the basis of complete rigidity at the connections. If the web thickness is such that

\[ w = 2.00 S/d^2 \]

then, theoretically at least, adequate elastic stiffness is assured.
(f) An end plate is preferable to the use of stiffeners as extensions of the column tension flange as was used in the test. (See types 8 and 14, Fig. 5).

(6) **TYPE 8B** (K L M)

(a) Adequate diagonal stiffening was used to prevent yielding due to shear force.

(b) Excellent agreement between theoretical and experimental moment-rotation curves was obtained. This rotation is somewhat greater (14%) than the equivalent length stiffness implied in computations of deflections in continuous structures.

(c) Good agreement between theory and experiment was also obtained for inelastic moment-rotation and moment-deflection curves when it was assumed that the moment was uniform over the equivalent length of the knee.

(d) These connections yield at a considerably lower load than predicted, probably due to residual stress and stress concentrations. However the ultimate strengths developed are all adequate.

(e) The half-depth stiffener as used in L is to be preferred, decreasing the small influence of residual stress. It is doubtful that K has adequate rotation capacity.

(f) L and M have excellent rotation capacities. It is doubtful if K is adequate.
(7) TYPE 15 (C)

(a) This knee would be designed for a non-uniform moment gradient but was tested with an equal moment gradient on each leg. This amounts to examining the connection under a second loading condition. Although it develops adequate strength at one end, it will not maintain a plastic hinge long enough for the development of yield or collapse strengths at the other end. Little improvement in this characteristic could be obtained with a more rigid system of lateral support, since as is evident from Table VIII column 9 the strength at section b-b was somewhat greater than that obtained in L with its efficient support system.

(b) The elastic average unit rotation is given approximately, by the expression given in 2(d) above.

(c) The connection demonstrates adequate shape factor.

(8) TYPE 16 (N)

The comment for this connection is similar to that for type 15 above.

B. Structural Behavior

(1) INITIAL YIELD STRENGTH

(a) A number of methods for specifying the initial yield strength from a moment-deformation curve were examined. The "structural yield" method is easy to use and gives the same result as the offset method based on a yield offset of .02% (column 4e table VIII).
(b) In general the first yielding (due to combined 
residual stresses and stress concentrations)
ocurred at about half the computed initial yield 
value (column 6B). At 75%, a departure from linear 
behavior could be detected, (column 6) and at about 
the computed value, "yield" occurred beyond which 
deflections of the structure would be affected, 
(column 6A). There were two principal exceptions, 
P and B.

(c) All of the connections except C and N yield at 
lower loads than would be predicted on the basis 
of initial yielding at the end of the rolled section 
(column 6C and 6D). However, at increased deforma­
tion the initial yield strength was developed in 
the rolled section......of importance when deter­
mining where the connection and rolled section 
shall be spliced.

(2) ULTIMATE STRENGTH

(a) All of the connections except B and P developed a 
carrying capacity considerably greater than the 
calculated initial yield moment (column 8A, Table VIII).

(b) Only the type 7 connection is unable to carry a 
haunch moment equivalent to the plastic hinge value.

(c) Five of the connections will not develop the full 
rolled beam strength at the haunch ends (column 9, 
connections B, C, J, N, and P). Thus caution must 
be exercised in proportioning haunched connections 
on the basis of simple plastic theory.

(d) If the influence of axial load is taken into account
in theoretical computations, then it may be stated with accuracy that all of the connections develop haunch moments sufficient to develop the yield strength of the rolled section at the splice. The influence of axial load may be neglected and the statement is still true for all but three of the connections.

(3) STIFFNESS

(a) Expressions for elastic stiffness of straight knees have been developed which give fair agreement with type 7 connections and good agreement with types 2 and 8B, (diagonally stiffened). The development also includes expressions for straight knees of the three types with equal depth and with dissimilar section depths.

(4) RESIDUAL STRESS

(a) In some cases being combined inseparably with stress concentration effects, residual stress accounts for a lowering of the initial yield strength. In flexural members the influence is not permanent and disappears after larger rotations are experienced.

(b) Residual stress was possibly a factor in bringing about the early collapse of the type 2B connection (B). The reason is that the haunch compression flange acts as a column and it appears that residual stress will certainly influence the
carrying capacity of compression members, although its influence on flexural members is negligible.

(5) **INELASTIC INSTABILITY**

(a) Plastic instability was involved in the collapse of every connection and in most cases brought about final collapse.

(b) Deformations increase rapidly once local buckling has occurred. In the case of straight knees in which it is possible to provide lateral support at the most important point, collapse does not occur when the first local buckling becomes evident, since the buckling is symmetric on both sides of the web. However when the second wave is formed together with web instability collapse follows rapidly. For connection L, however, this was well beyond the needed rotation capacity.

(c) The seriousness of local buckling is markedly reduced whenever lateral support can be placed at each point of expected yielding.

(6) **LATERAL SUPPORT**

(a) The flex-bar method of lateral support is to be preferred over the vertical guide system. The former simulates more precisely the situation encountered in practice.

(b) It is important to place lateral support at points of expected maximum stress. Whenever it is possible to do this at every maximum stress point then the seriousness of local buckling may be markedly reduced.
(c) A rather small force is required initially to prevent sidewise deformation. After local buckling this load increases rapidly.

(d) Compression flange lateral support should be provided at the center of built-up haunches and at the splice points between haunch and beam.

(e) Except for variations in cost, so far as plastic strength is concerned, effective lateral support is more important than the variations in fabrication details.

(7) ROTATION CAPACITY

(a) Depending on the proportions of the frame, for straight knees, each connection must have a rotation capacity after the plastic hinge value has been reached in order that the moment may be maintained while hinges are forming in other parts of the structure.

(b) In the case of a third-point-loaded beam attached to very stiff columns, a rotation capacity of about eight times the rotation at initial yield is required.

(c) With adequate lateral support the type 85 connections are satisfactory, while the behavior of type 2 is doubtful. Type 7 connections are unsatisfactory due to shear deformation.

(d) While some of the built-up knees have fair rotation capacities, most of them collapse very rapidly after first local buckling. This includes those that are well-supported laterally. Normally rotation capacity is not a requirement of haunched connections, however.
(e) Rotation capacity is measured by the ability of the knee to resist the effects of local buckling. Thick flanges and effective lateral support are most helpful.

(8) COST OF FABRICATION

(a) There is an almost linear relation between the time required for cutting and welding and the total length of haunch as measured along the neutral lines. Connections N and J are exceptions.

(b) Sniped, half-depth stiffeners should constitute an economical advantage over those of full-depth, and in the last analysis will always be less expensive than the complete stiffener welded to the web and to both top and bottom flange.
XI: ACKNOWLEDGEMENTS

The authors wish to express their appreciation to the sponsors of this research program. Acknowledgement is also expressed for the help and support received from members of the Lehigh Project Subcommittee (Welding Research Council), particularly that of its Chairman, Mr. T. R. Higgins. Mr. William Spraragen, Director, Welding Research Council, and Mr. LaMotte Grover, Chairman, Structural Steel Committee, have generously contributed their suggestions and criticisms.

This work has been carried out in the Fritz Engineering Laboratory of which Prof. Wm. J. Enay is Director. Acknowledgement is also due Mr. Jan M. Rusak, Research Assistant, for his tireless help in the early part of the test program. Professor C. D. Jensen has contributed his suggestions in parts of the manuscript. Thanks is expressed to Mr. Kenneth R. Harpel, foreman, and the staff of machinists and technicians in the Fritz Laboratory.
Fig. 5 Portal Frame Knees

Square Knees

Hunched Knees

Curved Knees
Connection Behavior Compared with BB13 Rolled Section (Based on Moment at the Knee).

Notes:
- Moments shown have not been corrected for increase in moment.
- L: local buckling.
Fig. 167: - Explanation of Comparisons of Table VIII

THEORETICAL MOMENTS

Initial Yield
$M_h = M_p$

Maximum observed "Yield"

Initial Yield (by eye)
First Yield Line

EXPERIMENTAL VALUES
<table>
<thead>
<tr>
<th>Connection</th>
<th>Type</th>
<th>1st Yield Line (Local Yielding)</th>
<th>Observed Initial Yield Moment at the Knee (in-kips)</th>
<th>Calculated Initial Yield Moment at the Knee (in-kips)</th>
<th>Max. Moment in Rolled Section (in-kips)</th>
<th>Max. Moment at the Knee (in-kips)</th>
<th>Max. Moment in Rolled Section (for BB 13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>163</td>
<td>460</td>
<td>369.476104</td>
<td>486</td>
<td>1.126</td>
<td>548</td>
</tr>
<tr>
<td>B</td>
<td>2B</td>
<td>305</td>
<td>618</td>
<td>380.600</td>
<td>348</td>
<td>2.10</td>
<td>728</td>
</tr>
<tr>
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* Point at which moment-rotation curve deviates from straight line.

1 0.920 is ratio based on 1190 in.-kips, the M_p of 8 WF 3I.