Constructional alloy steels - column behavior
(letter report) (may 1955)

Beedle, L. S.
1955
To: Members, Research Committee A

Re: Constructional Alloy Steels - Column Behavior

Gentlemen:

Last year we were asked by U. S. Steel Corp. to determine the column strength of one of their constructional alloy steels rolled to 2 3/4" diameter solid round. Since the compressive properties were not known, this seemed to us to be a nice opportunity to gain an additional correlation between the full cross-section strength (and average stress-strain diagram) and the strength of columns.

We therefore undertook the tests, they are now completed, and through the courtesy of U. S. Steel Corporation we are able to present a preliminary report of the results.

The Tests

The material was a constructional alloy steel of 90,000 psi minimum yield strength. As noted above, the material was rolled to 2 3/4" diameter bars. The bars were then heat treated at the mill by quenching and tempering. In order to minimize the eccentricity, the bars were cold straightened to closer than commercial tolerances, after which they were stress relief annealed to minimize residual stresses formed due to cold-straightening. It was suggested that the particular heat treatment accorded these bars would result in reasonably uniform properties across the section. If the properties were not uniform or if some residual stresses remained, an otherwise linear stress-strain diagram would show a proportional limit somewhat lower than would be observed in a tension coupon test.

By determining the tangent-modulus at various stress levels from the average stress-strain curve from cross-section tests and by use of the tangent modulus formula (1)

\[ \frac{P}{A} = \frac{\tau^2 E_t}{(KL/r)^2} , \]

a column curve should be obtained. The concentrically-loaded column tests should indicate how well actual column tests agree with the "column curve".

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The following tests were therefore carried out:

<table>
<thead>
<tr>
<th>No.</th>
<th>Test Description</th>
<th>Test Numbers</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Cross sections (2 3/4&quot; x 11&quot;)</td>
<td>Test Nos. C-1, C-2, C-9-2</td>
<td>Test Nos. C-9-1, C-9-3, C-4, C-6, C-7, C-8, C-10, C-11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Constructional Alloy Steel</td>
</tr>
<tr>
<td>1</td>
<td>Cross section as a comparison</td>
<td>Test No. C-3</td>
<td>A-7 Steel</td>
</tr>
<tr>
<td>8</td>
<td>Centrally-loaded columns (L/r = 57, 67, 100, 133)</td>
<td>Test Nos. C-9-1, C-9-3, C-4, C-6, C-7, C-8, C-10, C-11</td>
<td>Constructional Alloy Steel</td>
</tr>
<tr>
<td></td>
<td>Centrally-loaded column as a comparison test (L/r = 133)</td>
<td>Test No. C-12</td>
<td>A-7 Steel</td>
</tr>
<tr>
<td>1</td>
<td>Tension coupon (.505)</td>
<td>Test Nos. C-13, C-14</td>
<td>Constructional Alloy Steel</td>
</tr>
<tr>
<td>1</td>
<td>Tension coupon (.505)</td>
<td>Test Nos. C-13, C-14</td>
<td>Constructional Alloy Steel</td>
</tr>
<tr>
<td>2</td>
<td>Eccentrically-loaded columns (L/r = 38, ( \frac{E_C}{r^2} = 2.5, 3.5 ))</td>
<td>Test Nos. C-13, C-14</td>
<td>Constructional Alloy Steel</td>
</tr>
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</table>

Table 1 outlines the tests.

All tests, except the tension coupons, were run in the 800,000# screw-type Riehle machine at Fritz Laboratory. With the exception of the eccentric columns, all specimens were tested in the flat-end condition.

Precision of alignment was within about 3% for the cross-sections and about 5% for the columns. Cylindrical wedges were used for this purpose.

Investigations

We were able to call upon existing methods of analysis to predict the behavior of the centrally and eccentrically-loaded columns.

Application of the tangent-modulus principal to the problem has already been discussed. The application is approximate because every fibre of the cross section apparently does not have the same stress-strain characteristics. Members of the committee will remember this application in the case of A-7 WF shapes bent about the strong axis(2).

(2) "RESIDUAL STRESS AND THE COMPRESSIVE STRENGTH OF STEEL", by Huber and Beedle, Welding Journal 33(12), December 1954.
In the case of the eccentrically loaded columns, two comparisons are possible: (1) the secant formula, and (2) a plastic analysis to predict the maximum load. The latter, based on the same method as used to analyzed WF eccentric columns(3), was developed for round shapes by Mr. Ketter and Mr. Paris under the assumption of uniform properties and idealized stress-strain diagram. The extension has not yet been written up in report form.

Finally, due to the fact that our knife edges did not have sufficient capacity to test the columns in the pin end condition, it was necessary to test them "flat-ended". SR-4 gages were mounted at sufficient sections so that the points of inflection could be determined as the loading progressed and hence the effective length, KL, could be found.

Results

Table 1 shows some of the test results.

The various stress-strain curves are shown in Fig. 1. The tension stress-strain curve (.505) shows a yield strength that was about the average of the three compression cross-sections (127,300 psi). However, the proportional limit was considerably higher in the tension test.

There is some variation in the compressive properties for this material fabricated from the same lot of steel. (Max. = 133.1, Min. = 121.1 giving a 10% variation.) In all likelihood, a similar variation would have been found if three tension tests also had been made. It has been suggested that variations of this magnitude are normally to be expected and are within the specification limits for commercially heat treated material.

The carbon steel showed a yield strength in tension of 34,800 psi as determined by the producer and a compressive yield strength of 31,600 psi as determined in the laboratory cross-section test.

In Fig. 2 are shown the column curves derived from the three cross-section tests of the constructional alloy steel. Also shown by circles are the results of the column tests. The open circles are for an assumed value of \( K = 0.55 \). The solid circles are for the measured value of \( K \). The maximum strength of the two eccentric columns are also shown in Fig. 2. The secant formula solution ("Sec") and the theoretical maximum values ("Max") are indicated.

Cross-section C-9-2 and column C-9 were directly adjacent in the rolling. However, no record was available as to the position of the rest of the material. From the test results, it appears that column C-4 was adjacent to cross-section C-1 and column C-6 was adjacent to cross-section C-2.

In Fig. 3 all results are plotted on the same non-dimensional basis.

In Figs. 2 and 3 the results of the carbon steel column test (C-12) is also shown. Agreement with theory is excellent.

A typical load-vs-horizontal deflection curve is shown in Fig. 4. To the right is shown the curve of load-vs-vertical deflection. In Fig. 5 is shown a typical curve used to determine the points of inflection. We were surprised to find values as near to the theoretical .50 as were observed.

Summary

These tests indicate that the tangent modulus formula applied to the results of a cross-section test predicts somewhat higher strengths than obtained from corresponding column tests. But the agreement with theory is reasonable in the light of unavoidable out-of-straightness in members of this size.

The average proportional limit was about 86% of the compressive yield strength, and there was good agreement between the different tests. The proportional limit in tension was 96% of the yield value. There was marked difference in the plain carbon steel (proportional limit = 69% of yield strength for compression).

As an example, one approximate column curve for this material would take the form shown in the following equation if the procedure of Ref. 2 is used (based on the recommendation of Bleich[4]).

\[ \sigma_{cr} = \sigma_y - \left( \frac{\sigma_y - \sigma_p}{\pi^2} \right) \sigma_p \left( \frac{KL}{r} \right)^2 \left( \frac{KL}{r} < \frac{\sqrt{E}}{\sigma_p} \right) \]

\[ \sigma_{cr} = \frac{\pi^2E}{(KL)^2} \left( \frac{KL}{r} \geq \frac{\sqrt{E}}{\sigma_p} \right) \]

This is plotted in Fig. 3.

Since the material properties may be affected by size (due to heat-treating), some check tests on larger rounds are contemplated.

The secant formula is conservative by 50% (Test C-13). For this same test a more precise (ultimate) theory predicts the maximum strength within 3%.

The tests were performed and results reported here were analyzed by Yuzuru Fijita with the assistance of Tadahiko Kawai and Satoru Niimoto. Robert L. Ketter participated in the planning of the program and in the analysis. George Driscoll had charge of the arrangements for the tests.

We had the pleasure of working with Messrs. H. Malcolm Priest and John A. Gilligan both of U. S. Steel Corporation during the program.

Sincerely yours,

Lynn S. Beedle
Assistant Director

LSB:plt

CC: Mr. John A. Gilligan
    Members, Committee D
    Members, Executive Committee
<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Loading Condition</th>
<th>Length in.</th>
<th>Area in.</th>
<th>$L/r$</th>
<th>KL/r (K = 0.55)</th>
<th>KL/r (K = 0.50)</th>
<th>K Calculated from Measured Curvature</th>
<th>$P_{\text{max.}}$ (kips)</th>
<th>$\sigma_{\text{max}}$ (K/in²)</th>
<th>Remarks</th>
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<td>C-1</td>
<td>—</td>
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<td>6.06</td>
<td></td>
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<td></td>
<td></td>
<td>731</td>
<td>$\sigma_Y$ 121.1</td>
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<td>6.04</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>803</td>
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<td>—</td>
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<td></td>
<td></td>
<td></td>
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<td>191</td>
<td>$\sigma_Y$ 31.6</td>
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<td>Conc.</td>
<td>46 1/4</td>
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<td></td>
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<td>36.8</td>
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<td>36.9</td>
<td>33.5</td>
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<td>6.03</td>
<td></td>
<td>66.8</td>
<td>36.8</td>
<td>33.4</td>
<td>.53</td>
<td>744</td>
<td>123.4</td>
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<td>5.93</td>
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<td>100.3</td>
<td>55.2</td>
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<td>.50</td>
<td>570</td>
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<td>6.02</td>
<td></td>
<td>99.7</td>
<td>54.8</td>
<td>49.9</td>
<td>.52</td>
<td>550</td>
<td>91.4</td>
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<td>39 1/2</td>
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<td></td>
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<td>31.4</td>
<td>28.6</td>
<td>.53</td>
<td>728</td>
<td>121.1</td>
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<td>C-9-2</td>
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<td>11.0</td>
<td>5.99</td>
<td></td>
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<td>765</td>
<td>$\sigma_Y$ 127.7</td>
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<td></td>
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<td>.53</td>
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<td>118.8</td>
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<td>92</td>
<td>6.01</td>
<td></td>
<td>133.1</td>
<td>73.2</td>
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<td>.53</td>
<td>368</td>
<td>61.2</td>
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<td></td>
<td>132.9</td>
<td>73.1</td>
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<td>.55</td>
<td>372</td>
<td>61.8</td>
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<td>6.03</td>
<td></td>
<td>132.9</td>
<td>73.1</td>
<td>66.5</td>
<td>.51</td>
<td>176</td>
<td>29.2</td>
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<tr>
<td>C-13</td>
<td>Ecc.</td>
<td>26 1/4</td>
<td>6.02</td>
<td></td>
<td>38.0</td>
<td></td>
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<td>259</td>
<td>(\frac{\sigma}{\tau} = 2.85)</td>
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<tr>
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<td>26 1/4</td>
<td>6.02</td>
<td></td>
<td>38.0</td>
<td></td>
<td></td>
<td></td>
<td>223</td>
<td>(\frac{\sigma}{\tau} = 3.57)</td>
</tr>
<tr>
<td>C-9-3</td>
<td>Conc.</td>
<td>39 1/2</td>
<td>5.97</td>
<td></td>
<td>57.3</td>
<td>31.5</td>
<td>28.7</td>
<td>.53</td>
<td>709</td>
<td>118.8</td>
</tr>
<tr>
<td>C-9-2</td>
<td>—</td>
<td>11.0</td>
<td>5.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>765</td>
<td>$\sigma_Y$ 127.7</td>
</tr>
</tbody>
</table>
FIG. 1

STRESS-STRAIN DIAGRAMS

Constructional Alloy Steel
Tension Coupon

Carbon Steel Tension Coupon

C-1
C-2
C-3
C-9-2
COLUMN CURVES FOR CONSTRUCTIONAL ALLOY STEEL

- From K = 0.55 Assumed
- From K Determined by SR-4

Cross-section Tests
C-2
C-9-2
C-9-1
C-6
C-9-3
C-4
C-7
C-8
C-11
C-10
C-13
C-14

Predicted Max. Value
Carbon Steel C-12

Sec. Formula
Carbon Steel C-3
(K = 1.0 Assumed)

FIG. 2
COLUMN CURVE FOR CONSTRUCTIONAL ALLOY STEEL (NON-DIMENSIONAL)

FIG. 3

\[ \lambda = \frac{1}{W} \sqrt{\frac{\sigma}{E} \frac{KL}{r}} \]
CURVATURE OF C-10 (SR-4)
CONSTRUCTIONAL ALLOY STEEL

FIG. 5