Closure to "stability of frames under-primarily bending moments", Proc. ASCE, 90 (ST4), p. 255

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1964
Closure by Le-Wu Lu

Le-Wu Lu, A. M. ASCE - The discussions of the paper have included some alternate methods of analyzing the frame instability problem considered by the writer. These have contributed to the general understanding of instability problems.

Appreciation is expressed to Goldberg for his encouraging comment and for calling to the writer's attention a paper containing some tabulated functions which are useful in analyzing buckling problems.

It is difficult for the writer to recognize the significance of Guillard's discussion. In the original paper, it was clearly stated that "the loading system is intended to simulate approximately the axial loads and moments that occur in the lower stories of a single-bay, single-story frame." Obviously, it is not possible to study the exact behavior of a multi-story frame based on the results obtained for a single-story frame. However, the significant conclusions presented in the paper are believed to be valid for multi-story frames. Additional work is being carried out at Lehigh University to study the effect of primary bending moments on the instability of multi-story frames.

Guillard misunderstood completely the writer's statement with regard to Eqs. 21 and 27. It was pointed out that since the loading parameter N does not enter into the solution of Eq. 27, the manner in which the vertical loads are applied to the frame should be immaterial in analyzing the buckling condition of Eq. 21. This does not mean, however, that the buckling load is also independent of N. As illustrated in Fig. 6, the buckling load is determined by the point of intersection of Eqs. 26 and 27. Equation 26 does depend on N; therefore, different loading conditions result in different critical loads for the same frame.

The final point discussed by Guillard is concerned with the interpretation of the test results. In discussing the failure of test frame P-4, Guillard misused the information contained in Table 2. The yield load 2P given in the table is the load at which the maximum stress in the model frame reaches the yield stresses of the material, and is not the axial yield load of the cross section. The computations and the results presented by Guillard are therefore completely meaningless and have to be ignored. For frame P-4, antisymmetrical buckling occurred at a total load of 2P_{eqp} equal to $(6.46/14.12) \times 100 = 45.7\%$ of the yield load. This insures that the stresses in the frame were well below the yield stress; hence, failure was definitely due to elastic buckling.

The writer is in agreement with Davies that the effect of finite deformations should be considered in an instability analysis if the members deform excessively at the failure load. The use of a linear theory in which small displacement is assumed is sometimes justified for two reasons: (1) the mathematical analysis involved is considerably simpler and tabulated functions can be readily used; and (2) the theory often yields results which are sufficiently accurate for practical applications. The accuracy of the results presented in the paper for the symmetrical case should be checked by carefully controlled frame tests. Unfortunately, this has not yet been done. However, experiments on slender model frames failed by sideways buckling have been reported by Chwalla and Kollbrunner\(^33\). They observed that the buckling load could be predicted quite accurately by the linear theory even though the model frames had already deformed excessively before buckling occurred. In one frame, the observed mid-span deflection was as much as two-tenth of the span length. Deflection of this magnitude is comparable with the deflections computed by Davies and illustrated in Fig. 16.

The alternate approach discussed by Al-Sarraf for analyzing instability problems are noteworthy. The stability functions tabulated by Livesley and Chandler\(^13\) were used by the writer in his earlier studies on frame instability and found to be very convenient for practical computations. However, the use of the usual trigonometrical functions in expressing the final solutions would be more familiar to the general reader. Furthermore, the solutions so expressed can be readily compared with the results of some earlier studies.\(^8\),\(^9\).

Errata - Equation 13 should read

\[
\frac{H}{P} \left( -1 + \lambda_1 L_1 \cot \lambda_1 L_1 - \lambda_2 L_1 \tanh \frac{\lambda_2 L_2}{2} \right) - \frac{w L_1}{L} \left( 1 + \tanh \frac{\lambda_2 L_2}{2} \right) = 0 \quad \cdots (13)
\]

\(^{33}\) "Beitriœge zum Knickproblem des Bogentraegers und des Rahmens," by E. Chwalla and C. F. Kollbrunner, Der Stahlbau, Vol. 11, No. 12, 1938, p. 94